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Measuring the Welfare Costs of Inflation in a Life-cycle Model

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Measuring the Welfare Costs of Inflation in a Life-cycle Model*

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Abstract

In a neoclassical growth model with life-cycle households in which money is held to satisfy a cash-in-advance constraint, the optimal steady state inflation rate is not the Friedman rule – it is in excess of 20%. Lump-sum, age-independent money injections twist and flatten the lifetime profile of utility, making this profile look more like the one that would be chosen by a planner. The cost of monetary finance of lump-sum payments is the distortion introduced to the labor-leisure choice.

Key words: monetary policy, inflation, welfare costs, life-cycle model
JEL codes: E52, E31, E32, D58, D91

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1 Introduction

Measuring the welfare costs of inflation in dynamic general equilibrium models is, at this point, well-trod territory. By way of example, the early work of Cooley and Hansen (1989) placed the costs of a 10% inflation at around 0.5% of consumption, measured relative to the Friedman rule optimum. With but a few notable exceptions, discussed later in this introduction, estimates of the costs of inflation have been made within the representative agent framework. For many macroeconomic issues, the representative agent fiction is a useful one. For example, Ríos-Rull (1996) showed that for understanding aggregate business cycle phenomena, the life-cycle is largely irrelevant. The basic question asked in this paper is, "How do the costs of inflation change when agents differ by age?" For the model analyzed in this paper, a life-cycle version of the Cooley and Hansen, the answer is, "Quite a bit." More specifically, the optimal inflation rate (the one maximizing steady state lifetime utility of a newborn) is quite high – over 20%. Before giving some intuition for why high inflation is optimal, it will help to know the model's key features.

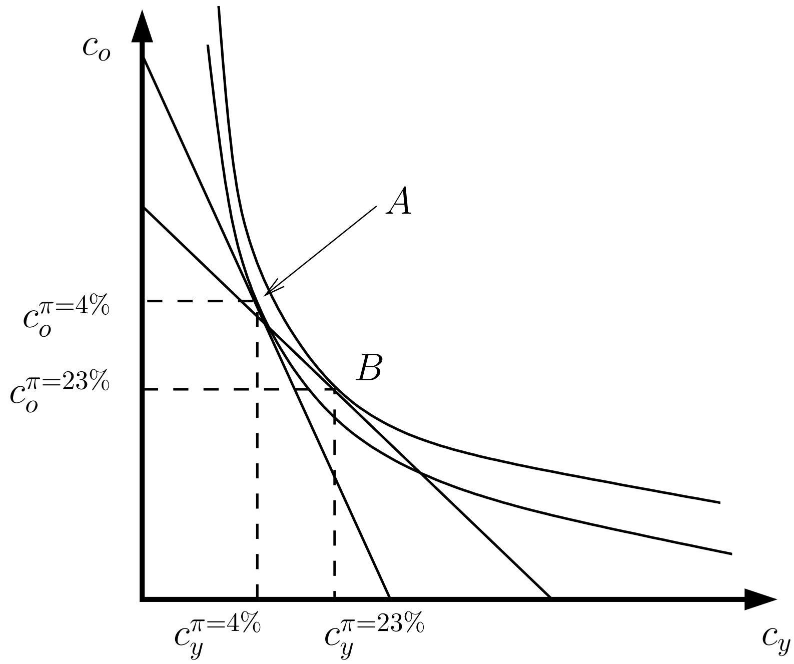
First, as in Cooley and Hansen (1989), money is held in order to satisfy a cash-in-advance constraint on purchases of the consumption good. Second, as is common in the monetary literature, money injections occur via lump-sum transfers (so-called helicopter drops of money). Third, individuals live exactly T periods; there is no random death as in Ríos-Rull (1996). Fourth, individuals start life with no capital (real assets), and must end their lives with non-negative capital holdings. Since there is no bequest motive, individuals will, in fact, end life with no capital. Between birth and death, individual are unconstrained with respect to their capital holdings, and so *may* go into debt if they wish. Finally, individuals start life with some real money balances. This feature is included so that there is not a 'trivial' reason for inflation to be welfare-improving: Suppose individuals have no initial real balances, then if there is no lump-sum transfer of money balances, the cash-in-advance constraint implies that individuals would be unable to purchase consumption in the first period of their lives. So that money is not simply created "out of thin air," it is assumed that agents end life with the same level of real balances with which they started.

Why is the optimal steady state inflation rate so high? A key feature of the calibrated model is that utility increases with age; with utility separable between consumption and leisure, both consumption and leisure also increase with age. The cash-in-advance constraint implies that in order to finance this increasing profile for consumption, real money balances must also rise with age. Inflation acts as a tax on old, rich agents, and the lump-sum injections of money transfer resources to young, poor agents. To understand why households prefer a flatter age-utility profile, consider the problem of a planner who maximizes a weighted sum of utility of all generations, including the unborn; this problem is discussed in more detail in Section 4. Suppose that the planner discounts each generation's lifetime utility using the same discount factor used by households; then the steady state profiles for utility, consumption and leisure are all constant provided preferences are separable between consumption and leisure. In other words, at least in steady state, flatter age-profiles for consumption, leisure and utility are desirable.

However, inflation introduces a distortion to the labor-leisure choice owing to the fact that current income cannot be spent on consumption goods in the same period in which the income is earned. There are, then, two effects associated with inflationary finance of lump-sum injections of money: (1) the flatter age-utility profile which raises steady state lifetime utility of the newly born, and (2) the distortions to the labor-leisure choice. For the calibrated model, the first effect dominates up to an annual inflation rate of 23%. For this inflation rate, the welfare gain computed in steady state for a newborn agent is 0.1% of consumption.

The intuition underlying the above results is summarized in Figure 1. In order to produce a relatively simple figure, the rich life-cycle structure of the model is reduced to consumption when young and consumption when old, and leisure is suppressed. Point A corresponds to the 5% inflation steady state (the U.S. historical average) and reflects the observation that in the model, consumption is rising with age. Point B, the 23% steady state, is obtained

Figure 1: Intuition



as follows. First, the combination of higher inflation and associated lump-sum transfers rotates the budget line since this higher inflation rate is associated with higher consumption when young, and lower consumption when old. Second, the budget line shifts in reflecting the deleterious effects of inflation on hours worked. As drawn, the new steady state consumption allocation places households on a higher indifference curve.

The model's results are driven by the fact that consumption, leisure and utility all increase with age. To understand why consumption rises over the life-cycle, it is easier to consider the non-monetary version of the model. When consumption and leisure are separable in preferences, as they are in the model, the intertemporal Euler equation governing real asset accumulation relates the growth rate of consumption over the life-cycle to the product of the discount factor and the gross real interest rate. The model is calibrated to a conventional value of the real interest rate, 4% per annum. The resulting value for the discount factor implies that the aforementioned product is greater than one, and so consumption rises with age. The intratemporal Euler equation then delivers a rising pattern to leisure, and finally utility.

Figure 1 also helps to explain why households do not achieve a greater degree of utility smoothing on their own. After all, households are free to go into debt. However, if they do so, they need to repay that debt at the real interest rate – in steady state, 4% per annum. The government, though, faces more of a one-for-one trade-off between young and old consumption. Roughly speaking, private households face a budget constraint associated with point A while the government faces a feasibility constraint associated with point B.

Transferring resources from the initial old to younger agents does not yield a Pareto superior allocation as can be seen in Figure 1. In particular, the utility of the initial old falls. To assess the magnitude of this problem, the transition path associated with a permanent, unanticipated change from 5% inflation, the U.S. average, to 23% is computed. A small fraction – only 14% – of those alive at the time of the policy change are made better off. It also takes a long time – over 40 years – before there is a measured welfare benefit. The overall welfare cost of such a policy change is found to be around 0.1% of consumption.

As mentioned earlier, there are two other notable papers that assess the importance of heterogeneity in measuring the costs of inflation: İmrohoroğlu (1992) and Erosa and Ventura (2002). The environment considered by İmrohoroğlu is one in which individuals hold money balances as a buffer against uninsurable income shocks (spells of unemployment). Her key result is that Bailey welfare triangles understate the costs of inflation by as much as a factor of 3. Erosa and Ventura's model has two types of agents, rich and poor. They allow for an endogenous cash-credit good distinction. Their model is calibrated to match observations for the United States, which implies that the poor purchase a greater proportion of their goods with cash, and so experience a greater burden of the inflation tax.

The remainder of the paper is organized as follows. The model is presented in Section 2, and calibrated in Section 3. Welfare results for the model economy can be found in Section 4. Transition dynamics are explored in Section 5. Section 6 concludes.

2 The Economic Environment

2.1 Households

At each date t a unit mass of identical individuals is 'born'. Each individual experiences exactly T periods of 'economic life.' The term *economic life* is used to refer to individuals who have entered the labor force and so participate in economic activity. Early childhood development and education are not considered here. Altruism between parents and their offspring is also suppressed. In order to analyze fairly realistic life-cycle dynamics, the lifespan T will be long. In the calibration section, a period will be specified as one quarter, and T will be set to 220, corresponding to 55 years of economic life.

Since individuals differ only as to their date of birth, individual-specific variables need to specify an individual's date of birth, and their current period of life. By way of example, n_t^i denotes the hours of work of an individual born at date t who is in their i^{th} period of life. In *calendar time*, these hours are supplied at date $t+i$.

Preferences for a member of generation t (that is, someone born at t) are given by:

$$E_t \sum_{i=0}^{T-1} \beta^i U(c_t^i, \ell_t^i), \quad \beta > 0.$$

The period utility function, U, is defined over consumption, c_t^i, and leisure, ℓ_t^i, and is assumed to possess standard properties. Future utility is discounted at the rate β.

Households face a number of constraints. To start, the nominal budget constraint is

$$\begin{aligned} P_{t+i}c_t^i + P_{t+i}[k_t^{i+1} - (1-\delta)k_t^i] + M_t^{i+1} = \\ W_{t+i}n_t^i + R_{t+i}k_t^i + M_t^i + X_{t+i}, \quad i = 0, \ldots, T-1. \end{aligned} \tag{1}$$

The right-hand side gives sources of funds. At age i, an individual supplies time to the market, n_t^i, earning a wage W_{t+i}. The second term on the right-hand side of Eq. (1) is capital income. The household starts period $t+i$ with real assets (or capital) k_t^i. It rents this capital for a nominal rental payment of R_{t+i}. The household also starts period $t+i$

with money balances, M_t^i. It receives a lump-sum transfers from the government, X_{t+i}.

The left-hand side of Eq. (1) represents uses of funds. The first term is nominal purchases of consumption goods. The household expends funds investing in capital, given by the second term on the left-hand side. Here, δ is the depreciation rate of capital. Negative investment is permitted and corresponds to a change in ownership in capital goods. Finally, the household departs period $t+i$ with nominal money balances M_t^{i+1}.

The household faces a standard cash-in-advance constraint:

$$P_{t+i}c_t^i \leq M_t^i + X_{t+i}, \quad i = 0, \ldots, T-1. \tag{2}$$

The term on the left-hand side of the cash-in-advance constraint is the value of consumption. These purchases are constrained by the sum of beginning-of-period money balances and the monetary lump-sum payment from the government, X_{t+i}.

The time endowment of an individual is normalized to unity; thus, labor and leisure must satisfy

$$\ell_t^i + n_t^i \leq 1, \quad i = 0, \ldots, T-1.$$

The only constraints that will be placed on capital holdings are that individuals start life with no capital, and they must end life with non-negative capital:

$$k_t^0 = 0, \quad k_t^T \geq 0. \tag{3}$$

$k_t^{i+1} < 0$ means that at age i a member of generation t went into debt.

The final two constrains are on money holdings. It is assumed that individuals start life with real balances, $\overline{m} > 0$, and must end life with the same level of money balances:

$$\frac{M_t^0}{P_{t-1}} = \overline{m}, \quad \frac{M_t^T}{P_{t+T-1}} \geq \overline{m}. \tag{4}$$

If $\overline{m} = 0$, then the cash-in-advance constraint, Eq. (2), would imply that positive first period of life consumption is feasible only if the transfer, X_t, is strictly positive. This transfer can be strictly positive only if money growth, and so inflation, is strictly positive. Absent positive initial money balances there would be a trivial reason for positive inflation to dominate the Friedman rule (deflate at the real interest rate) since this would be the only way for individuals to enjoy positive first period consumption.

The initial real balances could be thought of as a transfer made from a parent to an offspring, or as coming from income of a child prior to entering the labor force. The constraint on end-of-life real balances is imposed to conserve on aggregate private money balances (money balances are not being magically introduced through the endowment of the just-born).

Most of the constraints faced by an individual are satisfied owing to nonsatiation. The cash-in-advance constraint binds if inflation is sufficiently high to ensure that the return on capital exceeds that on money (so that no one would hold money as a store of value). It is assumed that this condition is, in fact, satisfied.

2.2 Goods Producing Firms

Firms face a sequence of static problems. Each period, the typical firm rents capital, K_t, and hires labor, N_t, to maximize real profits,

$$P_t F(K_t, N_t; z_t) - R_t K_t - W_t N_t, \tag{5}$$

where F is a standard constant-returns-to-scale production function and z_t is a shock to technology. Since F is constant-returns-to-scale, in equilibrium firms will earn zero profits. Consequently, there is no need to tackle the tricky issue of firm ownership when specifying the households' problems.

2.3 Government

The only role for government is to create (or destroy) money balances that are distributed lump-sum to households. The government's budget constraint is

$$X_t = \frac{(\mu_t - 1)M_t}{T} \tag{6}$$

where μ_t is the gross growth rate of money, M_t is aggregate money balances, and T is the number of generations alive at t. Consequently, each generation receives its 'share' of new money balances.

2.4 Competitive Equilibrium

A competitive equilibrium for this economy is defined in the usual way:

(1) Each member of cohort t chooses contingency plans for consumption, hours of work, capital and money holdings, so as to maximize lifetime utility taking as given the process generating prices and the evolution of the aggregate state.

(2) Firms maximize period-by-period profits taking as given prices.

(3) The government satisfies its budget constraint.

(4) Markets clear:

$$K_t = \sum_{i=0}^{T-1} k_{t-i}^i,$$

$$N_t = \sum_{i=0}^{T-1} n_{t-i}^i,$$

$$M_{t+1} = \sum_{i=0}^{T-1} M_{t-i}^{i+1},$$

$$\underbrace{\sum_{i=0}^{T-1} c_{t-i}^i}_{C_t} + \underbrace{\sum_{i=0}^{T-1} \left[k_{t-i}^{i+1} - (1-\delta)k_{t-i}^i\right]}_{I_t} = F(K_t, N_t; z_t)$$

In the market clearing conditions the summations are across individuals alive at date t. By way of example, c^i_{t-i} is the consumption at date t of a typical member of cohort $t-i$; at time t, this individual is aged i.

2.5 The Pareto Problem

For future reference, it will be useful to characterize the allocation that would be chosen by a planner. Attention will be focused on steady state life-cycle profiles; to ease notation, uncertainty is suppressed. Given a set of weights attached to each generation, $\{\omega_t\}_{t=-T+1}^{\infty}$, the planner chooses sequences for consumption, hours of work, and aggregate capital to maximize

$$\sum_{t=-T+1}^{-1} \omega_t \left[\sum_{i=T+t}^{T-1} \beta^i U(c^i_t, 1-n^i_t) \right] + \sum_{t=0}^{\infty} \omega_t \left[\sum_{i=0}^{T-1} \beta^i U(c^i_t, 1-n^i_t) \right]$$

subject to

$$\sum_{i=0}^{T-1} c^i_{t-i} + K_{t+1} = F\left(K_t, \sum_{i=0}^{T-1} n^i_{t-i} \right) + (1-\delta)K_t, \quad t = 0, 1, 2, \ldots$$

The first sum in the objective function corresponds to the remaining utility of generations born before $t=0$ while the second sum is utility of generations born at or after $t=0$. The constraint merely reflects feasibility.

Assume that the weight attached to generation t is given by $\omega_t = \beta^t$; in other words, the planner discounts utility of each generation by the same discount factor used by households. Taking first-order conditions and focusing on steady state, the following equations must be satisfied:[1]

$$U_\ell(c^i, 1-n^i) = U_c(c^i, 1-n^i) F_n(K, N) \tag{7}$$

$$1 = \beta \left[F_k(K, N) + 1 - \delta \right] \tag{8}$$

$$U_c(c^i, 1-n^i) = U_c(c^{i+1}, 1-n^{i+1}). \tag{9}$$

[1]See appendix A.3 for a detailed derivation.

Eq. (8) determines the modified golden rule capital stock; Eq. (7) corresponds to the intratemporal optimization condition associated with the non-monetary economy; and Eq. (9) says that the planner wishes to equate the marginal utility of consumption over the life-cycle. This last condition arises from the assumption that $\omega_t = \beta^t$.

When utility is separable between consumption and leisure, Eq. (9) implies that consumption is *constant* over the life-cycle. Eq. (7) then implies that labor is also constant over the life-cycle. Consequently, the utility-age profile will be flat.[2]

3 Calibration

The length of a period is set to one quarter, and individuals live exactly 55 years; thus, $T = 220$. By design, the calibration corresponds as closely as possible to Cooley and Hansen (1989).

The period utility function is

$$U(c, \ell) = \ln c + \omega \ln \ell.$$

The goods production function is

$$F(K, N; z) = zK^{\alpha} N^{1-\alpha}.$$

The parameters governing production are taken from Gomme and Rupert (2007). The capital share parameter, α, is set to 0.283 and corresponds to capital's share of income from the U.S. National Income and Product Accounts. The technology shock, z_t, follows a first-order autoregressive process,

$$\ln z_t = \rho \ln z_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2).$$

Over the sample 1954–2001, Gomme and Rupert estimate $\rho = 0.9641$ and $\sigma_\epsilon^2 = 0.008164$.

[2] İmrohoroğlu, İmrohoroğlu and Joines (1995) consider a planner's problem in which the planner chooses among steady state profiles to maximize lifetime utility subject to feasibility. Solving this problem leads to a declining life-cycle profile for consumption and a rising profile for hours worked. The difference is that İmrohoroğlu *et al.* in effect set $\omega_t = 1$ for all t.

Table 1: Estimates of the Money Growth Process

	Currency
$\overline{\mu}$	1.012362
ψ	0.8327293
Standard Error	(0.0382036))
σ_ξ^2	0.00446666
Sample	1954Q1–2003Q2

The depreciation rate for capital, δ, is set to 0.01777, implying an annual depreciation rate of 6.9%, a value that corresponds closely to the average depreciation rate implicit in the capital stock and depreciation data reported by the Bureau of Economic Analysis.

Money growth also follows a first-order autoregressive process,

$$\mu_t = \psi\mu_{t-1} + (1-\psi)\overline{\mu} + \xi_t, \quad \xi_t \sim N(0, \sigma_\xi^2)$$

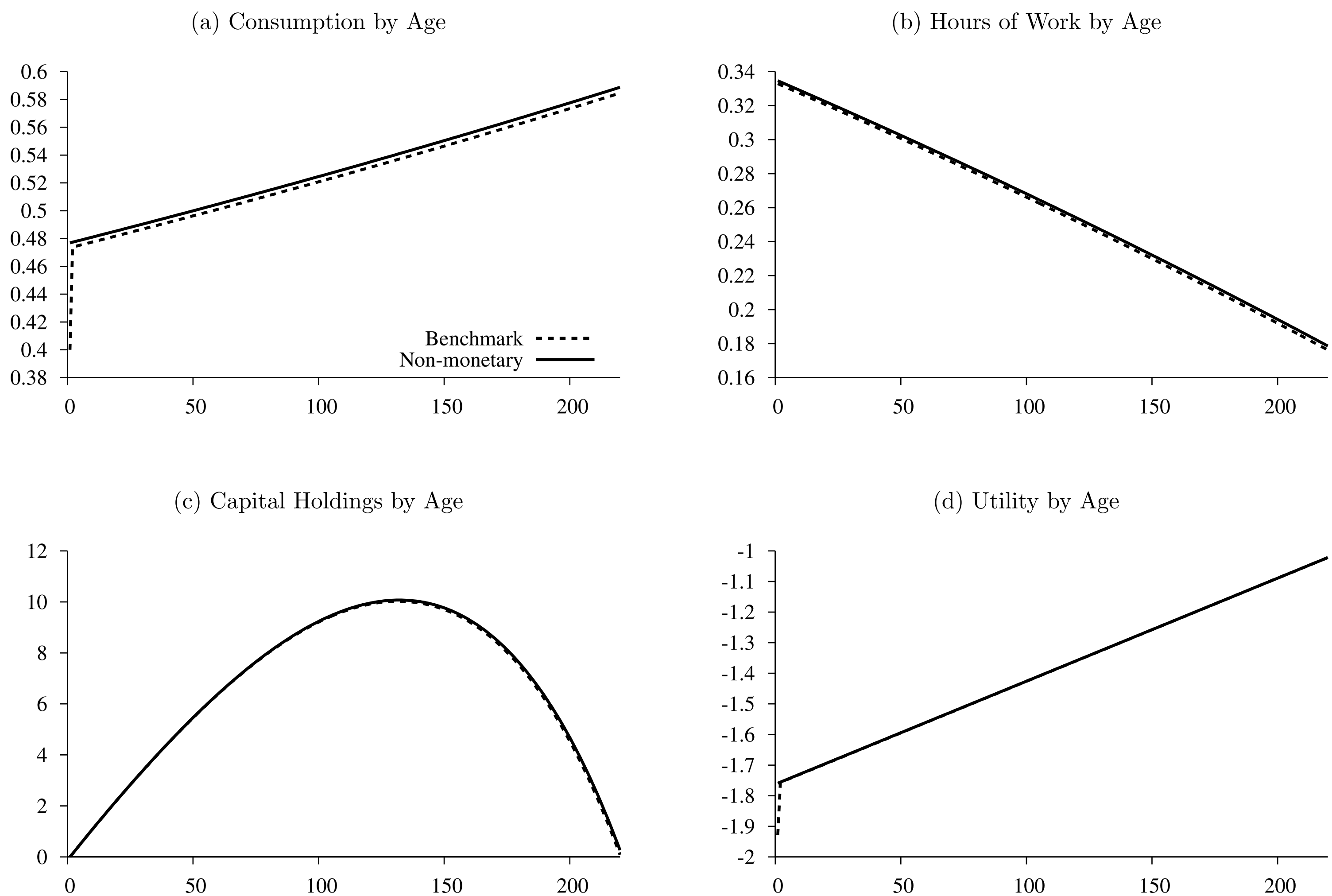
where $\overline{\mu}$ is the long run money growth rate. The parameters governing the behavior of money growth are estimated from U.S. data on per capita currency. These parameter estimates are summarized in Table 1. Over the sample period, average (quarterly) money growth has been fairly low. The stochastic processes for the technology shock and money growth are assumed to be uncorrelated. Running SUR on the Solow residual and per capita currency growth gives similar parameter estimates to the above; the innovations have a correlation of 0.0936 which is not significantly different from zero.

There are two preference parameters that have yet to be assigned values: the discount factor, β, and the leisure weight, ω. These parameters are set such that in steady state: (1) the real interest rate is 4% per annum which is a typical value used in the real business cycle literature;[3] and, (2) households work, on average 0.255 of the time, a value consistent with time-use surveys; see Gomme and Rupert (2007).

Finally, $\overline{m}$, initial and final real money balances, are set to 0.4, which constrains first pe-

[3]The optimal inflation rate and size of the welfare benefit of positive inflation both increase with the real interest rate.

Figure 2: Steady State



(a) Consumption by Age

(b) Hours of Work by Age

(c) Capital Holdings by Age

(d) Utility by Age

Table 2: Model Parameter Values

Preferences		
β	0.9911	discount factor
ω	2.5003	labor-leisure weight
σ	1.0	coefficient of relative risk aversion
Technology		
α	0.283	capital's share of income
δ	0.0178	depreciation rate of capital
ρ	0.9643	technology shock, autoregressive parameter
σ_ϵ	0.0082	standard deviation of innovation to technology shock
Money Growth		
ψ	0.8329	autoregressive parameter
$\overline{\mu}$	1.0124	long run annual money growth rate
σ_u	0.0045	standard deviation of innovation to money growth
Other		
T	220	number of periods of life
Calibration Targets		
$\overline{h}$	0.255	average hours worked
$\overline{r}$	0.01	real interest rate (quarterly)

riod consumption; see Figure 2. This value for $\overline{m}$ is roughly 84% of first period consumption for the non-monetary version of the model. The optimal inflation rate increases as $\overline{m}$ falls, although the results are not too sensitive to this parameter.

The values of the parameters for the calibration are summarized in Table 2.

3.1 Steady State

The age-profiles of consumption, hours of work, capital (real assets) and utility are graphed in Figure 2, along with the profiles corresponding to a non-monetary version of the model (in which case all goods are effectively credit goods). The non-monetary steady state is presented to verify that the introduction of money into the life-cycle model does not severely alter the nature of the model's steady state.

The consumption profile rises monotonically with age. It is, perhaps, easiest to understand the shape of this profile in the non-monetary version of the model. In this case, one of the Euler equations is

$$U_c(c_t^i, 1 - n_t^i) = \beta E_{t+i} \left\{ U_c(c_t^{i+1}, 1 - n_t^{i+1})[1 + r_{t+i+1} - \delta] \right\}.$$

When preferences are logarithmic, in steady state this equation reads

$$\frac{c^{i+1}}{c^i} = \beta[1 + r - \delta]. \tag{10}$$

The term in square brackets is the gross real interest rate which is fixed in the calibration process (for the monetary steady state). It turns out that the product of the discount factor and the gross real interest rate is larger than unity implying that individuals will chose a path for consumption that grows over their lifetimes.

The profile for real money balances (not in Figure 2) corresponds closely to that of consumption owing to the cash-in-advance constraint, and so also rises over the life-cycle. While capital holdings are not constrained to be non-negative, households nonetheless *choose* not to go into debt, saving until around chronological age 55 years (age 35 in the model, or $i = 140$), after which they dissave. Since there is no bequest motive, individuals will end their lives with no real assets. The age-profile of utility rises throughout the life-cycle.

The fact that the monetary and non-monetary steady states are so close to each other suggests that money is not distorting individual behavior too much. This observation is not too surprising in light of the modest money growth (and consequently inflation) rates; in the model, money growth is calibrated to the growth rate of U.S. currency per capita, 5% per annum.

3.2 Business Cycle Moments

Another litmus test for the model is whether its predictions for business cycle moments are similar to those reported in the literature. Table 3 reports business cycle moments for the U.S. economy, the monetary model, and the non-monetary model.

There are two important points. First, the model's performance (whether monetary or

Table 3: Selected Moments

	Standard Deviation	Cross Correlation of Real Output With								
		x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
U.S.										
Output	1.92	0.10	0.35	0.61	0.84	1.00	0.84	0.61	0.35	0.10
Consumption	0.85	0.19	0.39	0.58	0.74	0.79	0.70	0.55	0.37	0.18
Investment	4.70	−0.20	−0.03	0.22	0.49	0.73	0.82	0.80	0.68	0.49
Hours	1.76	−0.13	0.09	0.35	0.62	0.84	0.89	0.80	0.63	0.42
Productivity	1.04	0.40	0.49	0.54	0.50	0.41	0.03	−0.25	−0.44	−0.52
Capital	1.09	0.05	−0.04	−0.13	−0.21	−0.26	−0.30	−0.32	−0.30	−0.26
Monetary										
Output	1.65	0.08	0.24	0.45	0.70	1.00	0.70	0.45	0.24	0.08
Consumption	0.67	−0.07	0.07	0.26	0.51	0.81	0.65	0.50	0.37	0.25
Investment	7.07	0.14	0.29	0.47	0.70	0.96	0.64	0.37	0.16	−0.01
Hours	0.85	0.16	0.31	0.50	0.72	0.98	0.64	0.36	0.14	−0.03
Productivity	0.83	0.00	0.16	0.38	0.65	0.98	0.74	0.52	0.34	0.19
Capital	0.43	−0.42	−0.32	−0.17	0.04	0.32	0.51	0.61	0.64	0.63
Non-monetary										
Output	1.65	0.09	0.25	0.46	0.70	1.00	0.70	0.46	0.25	0.09
Consumption	0.56	−0.08	0.09	0.31	0.59	0.94	0.76	0.60	0.45	0.31
Investment	6.99	0.16	0.31	0.49	0.72	0.99	0.65	0.38	0.17	0.01
Hours	0.84	0.18	0.33	0.51	0.73	0.98	0.64	0.36	0.15	−0.02
Productivity	0.83	0.01	0.18	0.39	0.66	0.98	0.74	0.54	0.36	0.21
Capital	0.43	−0.42	−0.33	−0.17	0.04	0.33	0.52	0.62	0.66	0.65

Notes: Moments for the U.S. economy are based on Hodrick-Prescott filtered moments, are taken from Gomme and Rupert (2007) and correspond to the period 1954:I through 2004:II. Model moments are averages over 1,000 simulations of the model with each simulation of length 201 (the same number of observations as for the U.S. data), and Hodrick-Prescott filtered. Output is measured by private GDP, net of housing income flows; consumption by private consumption of non-durables and services, again net of housing income flows; investment by private non-residential fixed investment; hours by private non-farm hours; productivity is output divided by hours; and capital is non-residential fixed capital with quarterly data constructed from the annual capital stock and quarterly investment flows as described in Gomme and Rupert.

non-monetary) is on par with that of standard real business cycle models (with a representative, infinitely lived agent). This finding should not be too surprising since Ríos-Rull (1996) found that an annual version of the life-cycle model generated business cycle moments similar to that of the standard real business cycle model.

Second, adding money and money growth fluctuations has a fairly minor impact on the model's predictions for business cycle fluctuations. Cooley and Hansen (1989) made a similar observation for a representative, infinitely lived agent model.

In summary, nothing in this section suggests that there is anything odd about the monetary model.

4 Welfare Costs of Inflation

4.1 Lifetime Utility in Steady State

The criterion used for evaluating the desirability of money growth (or inflation) rates is steady state lifetime utility of newborns. This is the same criterion used in, for example, the social security literature; see İmrohoroğlu *et al.* (1995) and Huggett and Ventura (1999), among others. This welfare measure corresponds exactly to the across-steady-state comparisons of Cooley and Hansen (1989). Since steady state decisions differ across money growth rates, index these decision rules by μ. Steady state lifetime utility, conditional on money growth μ, can be expressed as:

$$V(\mu) \equiv \sum_{i=0}^{T-1} \beta^i U[c^i(\mu), \ell^i(\mu)].$$

Figure 3c plots $V(\mu)$ against a range of money growth rates. Remarkably, steady state lifetime utility is maximized at a money growth (inflation) rate of 23% per annum.[4] By way of contrast, in models with an infinitely-lived representative agent, like Cooley and Hansen

[4] A natural question is whether the extremely high inflation rate is the result of a programming error. While computing the steady state is a computationally demanding task, verifying it is not. In particular, the steady state quantities can be dumped into a file, imported into a spreadsheet, and the Euler equations and other constraints can be verified by hand. Doing so reveals no errors in computing steady state.

(1989), steady state utility is maximized by setting $\mu = \beta$ which implies a negative (net) money growth rate. Such a money growth rate results in a zero nominal interest rate, a result known as the 'Friedman rule.'

As seen elsewhere in the literature, higher inflation (money growth) is associated with diminished aggregate market activity; see Figures 3a and 3b. For example, Cooley and Hansen (1989) find that an increase in the annual inflation rate from 0% to 10% lowers aggregate output, consumption and hours by 2.3%; for the model, real activity falls by 1.7%.

That the steady state, lifetime utility-maximizing money growth rate is so high is even more surprising given the similarity in the life-cycle profiles of consumption and leisure (hours of work) across the monetary and non-monetary models' steady states presented in Figure 2. That is, money growth does not introduce a substantial distortion into the steady state of the model.

To understand the results regarding the optimal money growth rate, recall that when utility is separable between consumption and leisure, the solution to the Pareto problem in Section 2.5 implies that consumption, leisure and utility are *constant* over the life-cycle. Figure 4 presents life-cycle profiles for the calibrated money growth rate (5%) and the optimal money growth rate (23%). Notice that the higher money growth (inflation) rate twists the utility profile, making it flatter, bringing it closer to the solution to the Pareto problem.

Why should higher inflation lead to improved utility-smoothing over the life-cycle? Recall that consumption (and, via the cash-in-advance constraint, real money balances) grows over the life-cycle. Consequently, older agents pay a higher inflation tax than younger agents – but the proceeds of the inflation tax are rebated independent of age. Figure 4d shows that for the optimal inflation rate, net taxes paid – that is, the inflation tax paid less the lump-sum transfer – are positive for old households while young households receive transfers on net. In other words, inflation is a means of transferring resources from old, rich households to young, poor ones.

Of course, there is a cost to inflation. As is standard in cash-in-advance models, inflation

Figure 3: Steady State Values Plotted Against Money Growth

(a) Aggregate Output and Consumption

(b) Aggregate Hours

(c) Lifetime Utility

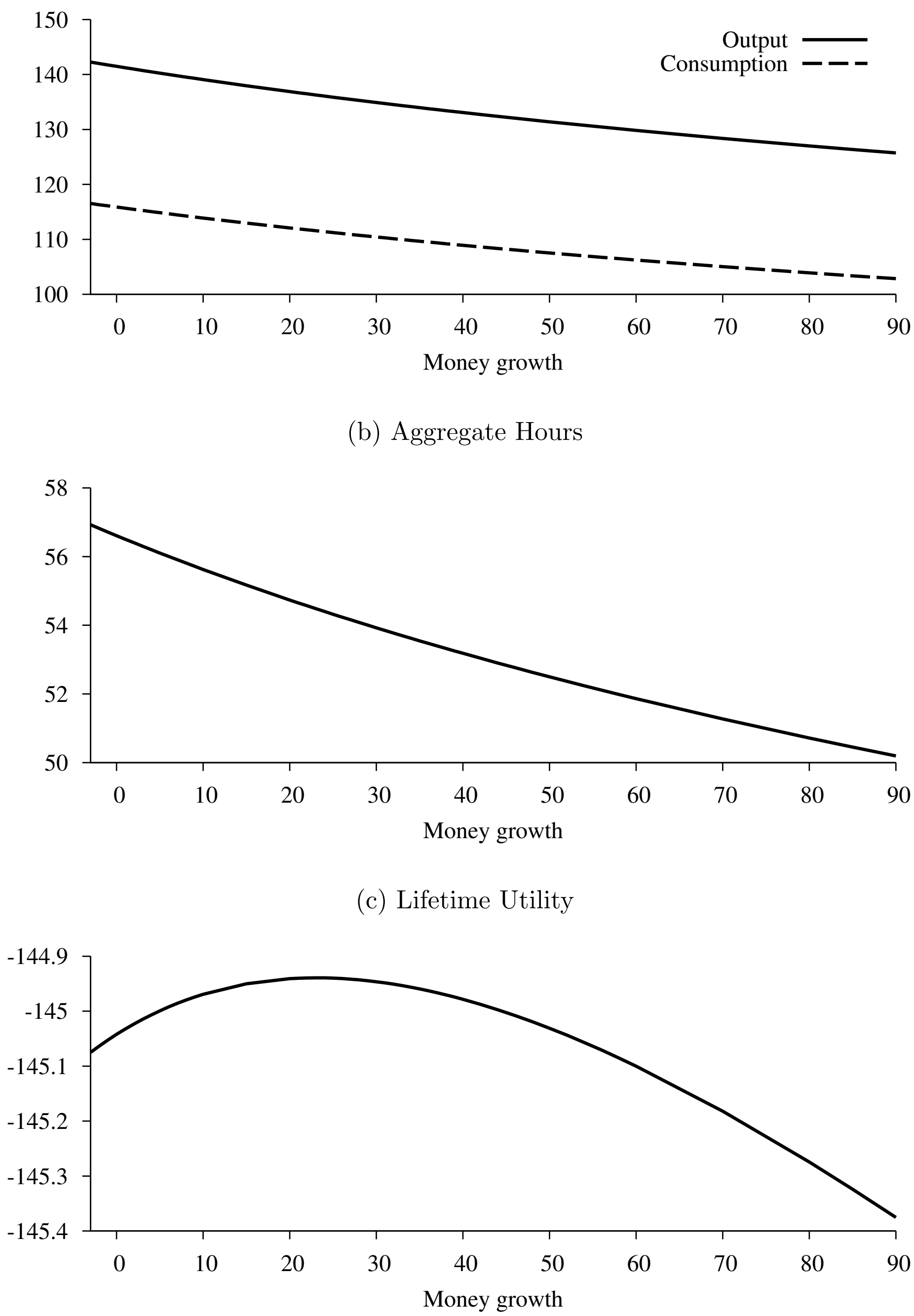
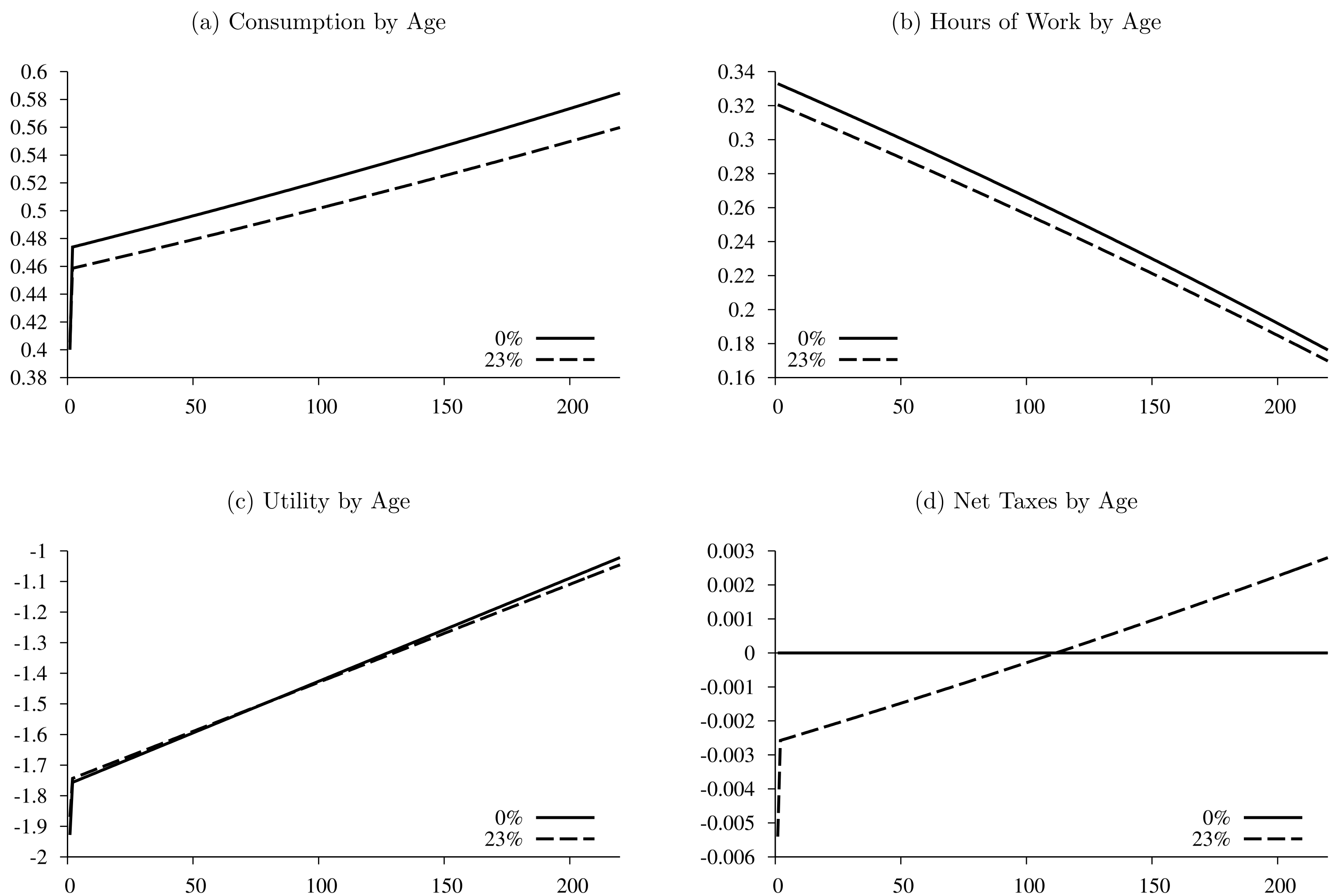


Figure 4: Utility-Maximizing Money Growth (23%) vs. 0%



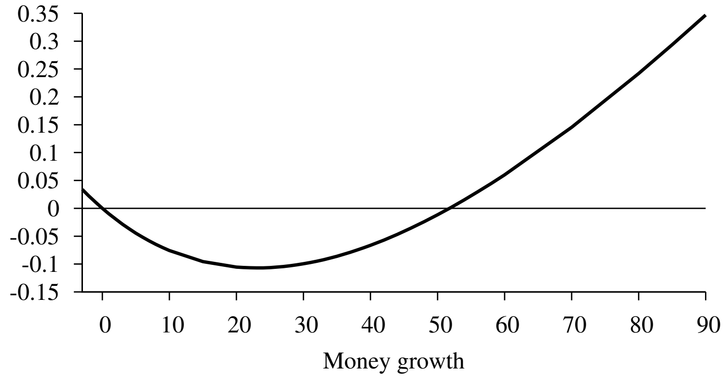
introduces a distortion into the labor supply decision since cash earned in the current period cannot be spent until the subsequent period when inflation has eroded its purchasing power. Figure 4a shows that, apart from the first period of life, the entire age profile of consumption falls with inflation while Figure 4b shows that hours of work similarly falls at all ages. That utility rises early in the life-cycle means that the increase in leisure (decline in labor) more than offsets the decline in consumption.

One might think that households should be able to achieve, on their own, any utility-smoothing that they desire since they are free to go into debt. It turns out that households *choose* not to go into debt. However, suppose that they did go into debt; eventually, they would have to repay this debt including interest. In a sense, the set of taxes and transfers engineered by the government via money creation acts somewhat like the government borrowing on behalf of the young at essentially a zero real interest rate. That is to say, the government faces a different feasibility constraint than that implied by the sequence of budget constraints confronting households.

Since the results in this paper are being driven by the rising life-cycle profile for consumption and leisure, it is tempting to think that one can flatten out these profiles by simply adjusting the discount factor, β. Here, the discipline of general equilibrium modeling comes into play: the discount factor is not a 'free parameter;' it is calibrated along with other parameters. Of course, one could choose a different calibration. In particular, calibrating to a lower real interest rate will increase the calibrated value of β. The lower real interest rate target requires a lower marginal product of capital, and so a higher capital stock. In turn, households must be more patient in order to hold more capital, particularly at a lower real interest rate. Both of these factors – lowering the steady state real interest rate and raising the discount factor – will flatten the life-cycle profile of consumption as suggested by Eq. (10). However, the real interest rate required to make the life-cycle profile for consumption essentially flat is near zero – an empirically implausible value.[5]

[5]Another means of flattening the life-cycle profile of consumption is to introduce retirement.

Figure 5: Welfare cost (benefit if negative) of various steady state money growth rates



4.2 Welfare

The next task is to obtain a 'unit free' measure of how agents care about alternative inflation (money growth) rates. A common approach in the literature is to find an 'equivalent variation payment' – that is, how much consumption must be given to agents to make them indifferent between two alternative money growth rates.

Welfare costs are expressed relative to a zero inflation rate. Let $V(\mu_0)$ denote the lifetime utility associated with a zero money growth rate. The welfare cost of inflation is given by the age-independent value of $\lambda(\mu)$ that satisfies

$$\sum_{i=0}^{T-1} \beta^i U[(1+\lambda(\mu))c^i(\mu), \ell^i(\mu)] = V(\mu_0).$$

Table 4 and Figure 5 summarize the welfare calculations. The welfare-maximizing money growth rate associated with the welfare cost calculation above conforms with the money growth rate that maximizes lifetime utility. The largest welfare benefit (i.e., negative welfare cost) occurs around 23% annual money growth rates. The welfare benefits are not insignificant – around 0.1% of consumption.

Table 4: Welfare Costs of Inflation

Inflation	V	W
−3.0	−145.0748	0.0339
−2.0	−145.0633	0.0220
−1.0	−145.0525	0.0107
0.0	−145.0422	0.0000
1.0	−145.0325	−0.0101
2.0	−145.0234	−0.0195
3.0	−145.0148	−0.0284
4.0	−145.0068	−0.0368
5.0	−144.9993	−0.0446
6.0	−144.9923	−0.0518
7.0	−144.9858	−0.0586
8.0	−144.9798	−0.0648
9.0	−144.9743	−0.0706
10.0	−144.9692	−0.0758
15.0	−144.9501	−0.0956
20.0	−144.9408	−0.1054
21.0	−144.9399	−0.1062
22.0	−144.9394	−0.1067
23.0	−144.9393	−0.1069
24.0	−144.9394	−0.1068
25.0	−144.9398	−0.1063
26.0	−144.9406	−0.1056
27.0	−144.9416	−0.1045
28.0	−144.9429	−0.1031
29.0	−144.9445	−0.1015
30.0	−144.9464	−0.0995
35.0	−144.9594	−0.0860
40.0	−144.9783	−0.0663
50.0	−145.0310	−0.0116
60.0	−145.1000	0.0601
70.0	−145.1821	0.1455
80.0	−145.2746	0.2418
90.0	−145.3754	0.3470

Notes: V is lifetime utility. W is the welfare cost as measured by the constant fraction of consumption that must be added in each period of life to leave a newborn indifferent between a particular inflation rate and 0%.

5 Transition Dynamics

As in the textbook overlapping generations model, in considering a policy change, one must take into account what happens to the initial old, not just the newborns. The problem with simply switching from the 5% steady state to the 23% steady state is that older agents are made worse off. However, it is possible that gains of the younger agents are sufficiently large that they could 'bribe' the older agents to accept this policy change. To investigate this possibility, the welfare costs associated with the transition path following a permanent, unanticipated increase in the money growth rate from 5%, the U.S. average, to 23% is computed.

The welfare cost/benefit of this policy change is computed as follows. Let $t = 0$ be the date of the policy change. For agents born at $t \geq 0$, compute the fractional change in consumption that leaves them indifferent between the old and new regime, as in Section 4.2. For agents part way through their life-cycles at $t = 0$, compute the constant fraction of consumption each period $t \geq 0$ that leaves them indifferent between the two policies. To obtain an overall metric of welfare at $t \geq 0$, add all of the compensating variation changes in consumption for that date and divide by total consumption at that date. This welfare metric corresponds closely to that used in the steady state analysis, and is computed for each period following the policy change.

The transition path is computed using decision rules log linearized around the 23% inflation steady state, but with initial conditions given by the 5% inflation steady state. Figure 6 gives the effects on overall economic activity, where the responses are expressed as a percentage deviation from the 5% steady state. As in steady state, economic activity is depressed by this increase in the inflation rate. Most of the transition dynamics have played out after 30 quarters ($7\frac{1}{2}$ years).

Figure 7a plots lifetime utility of generations that experience the change in monetary policy (the horizontal axis gives the birth date of each cohort). Generations born prior to the policy change at $t = 0$ experience part of their lifetime under 5% money growth, then the remainder with 23% money growth which explains why lifetime utility of generations $t < 0$ changes (in particular, the horizontal axis does *not* measure calendar time). Figure 7a shows that over 86% of the generations that are alive at the time of the policy change are made worse off. In fact, it is only agents born roughly 30 quarters prior to the change who are made better off. Consequently, as reflected in Figure 7b (where the horizontal axis is in calendar time), there is an initial welfare *cost* of switching to an inflation rate of 23% – roughly 0.3% of aggregate consumption in the period that the policy is implemented. It also takes a fairly long time – over 120 quarters, or 40 years – before a welfare benefit is measured. The welfare benefit eventually overshoots its long run value of roughly 0.06% of aggregate consumption.[6]

The overall welfare cost of the policy change analyzed in this section is computed as follows. Calculate the present value of all of the equivalent variation payments (as described above) where the present value is computed using the discount factor of households. Similarly calculate the present value of aggregate consumption. The welfare cost is computed as the present value of the equivalent payments divided by the present value of aggregate consumption. Computed this way, the welfare cost of the proposed policy change is nearly 0.1% of the present value of consumption.

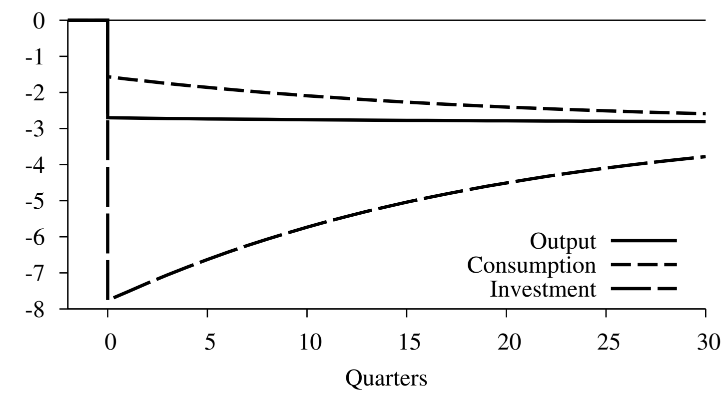
6 Conclusion

Of central interest to this paper is the curious result that in a life-cycle version of the neoclassical growth model used by Cooley and Hansen (1989), an inflation rate in excess of 20% maximizes steady state lifetime utility of newborns. Inflation has two effects. First, through the cash-in-advance constraint, inflation distorts the labor-leisure choice, an effect that necessarily reduces utility. Second, inflation transfers resources from old, rich households to young, poor ones. The reason behind this second effect is that real money balances rise with

[6]The long run welfare benefit is smaller than reported in the steady state analysis owing to the difference in the reference inflation rate (0% in the steady state analysis, 5% for the transition).

Figure 6: Transition Dynamics Following a Permanent, Unanticipated Change in Money Growth From 5% to 23% Per Annum, Occurring at Date 0

(a) Output, Consumption and Investment



(b) Hours and Productivity

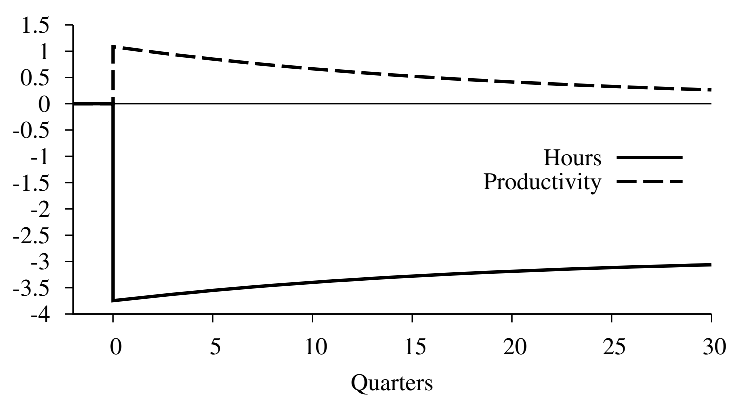
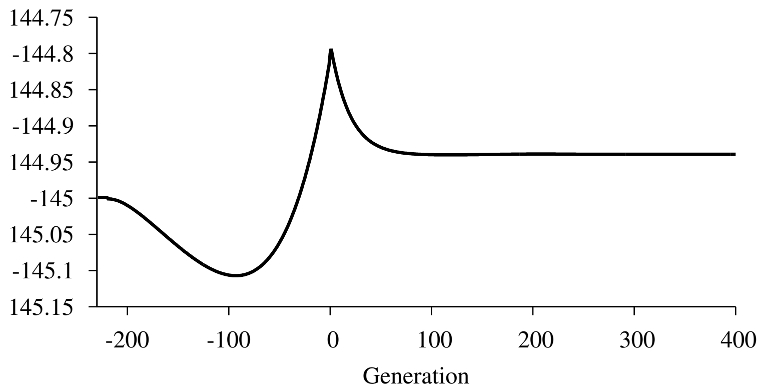
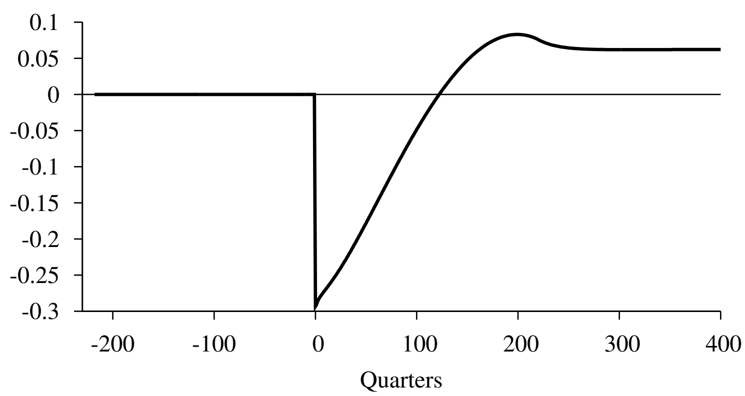


Figure 7: Transition Dynamics Following a Permanent, Unanticipated Change in Money Growth From 5% to 23% Per Annum, Occurring at Date 0

(a) Lifetime Utility



(b) Welfare Benefit



age – a consequence of the cash-in-advance constraint and an increasing age-consumption profile – which means that the burden of the inflation tax falls more heavily on old, rich agents while the lump-sum transfers are age independent. This second effect flattens the profiles for consumption, leisure and utility, making these profiles more like those chosen by planner who discounts utility of generations using the same discount factor as used by households.

Switching from the steady state allocation associated with 5% money growth to the one that maximizes steady state lifetime utility of newborns, 23%, does not deliver a Pareto superior allocation since the old are made worse off. To evaluate the magnitude of this problem, the transition between the 5% and 23% steady states was computed. This exercise revealed: (1) the vast majority of those alive at the time of the change are made worse off; (2) on impact, the welfare loss is 0.3% of consumption; and (3) it takes 40 years before an aggregate welfare benefit is recorded. The overall welfare cost of this policy change is just over 0.1% of consumption.

The results in this paper should have application to other taxes. Specifically, it is well known that there is an equivalence between the inflation tax, the labor income tax, and a consumption tax. In light of the results in this paper, a consumption tax or a labor income tax used to effect transfers of resources from the old to the young would also be found to improve steady state lifetime utility of newborns.

A Technical Appendix

A.1 Monetary Model

A.1.1 Household's Problem

The household's Bellman equation is:

$$
\begin{aligned}
V(k_t^i, M_t^i; i) \equiv \max \Bigg\{ & U(c_t^i, 1 - n_t^i) + \beta E_{t+i} V(k_t^{i+1}, M_t^{i+1}; i+1) \\
& + \Lambda_{1t}^i \Big[W_{t+i} n_t^i + \left[(1-\delta) P_{t+i} + R_{t+i}\right] k_t^i + M_t^i + X_{t+i} \\
& \quad - P_{t+i} c_t^i - P_{t+i} k_t^{i+1} - M_t^{i+1} \Big] \\
& + \Lambda_{2t}^i \left[M_t^i + X_{t+i} - P_{t+i} c_t^i \right] \Bigg\}.
\end{aligned}
$$

The choice variables are: c_t^i, n_t^i, k_t^{i+1} and M_t^{i+1}. Keep in mind the boundary conditions, Eqs. (3) and (4). The relevant Euler conditions are:

$$
\begin{aligned}
c_t^i : &\quad U_c(c_t^i, 1 - n_t^i) = P_{t+i} \left[\Lambda_{1t}^i + \Lambda_{2t}^i \right], \quad i = 0, \ldots, T-1 \\
n_t^i : &\quad U_\ell(c_t^i, 1 - n_t^i) = \Lambda_{1t}^i W_{t+i}, \quad i = 0, \ldots, T-1 \\
k_t^{i+1} : &\quad \beta E_{t+i} \Lambda_t^{i+1} \left[(1-\delta) P_{t+i+1} + R_{t+i+1} \right] = \Lambda_{1t}^i P_{t+i}, \quad i = 0, \ldots, T-2 \\
M_t^{i+1} : &\quad \beta E_{t+i} \left(\Lambda_{1t}^{i+1} + \Lambda_{2t}^{i+1} \right) = \Lambda_{1t}^i, \quad i = 0, \ldots, T-2
\end{aligned}
$$

These equations, along with the budget constraint, Eq. (1), cash-in-advance constraint, Eq. (2), and the boundary conditions characterize the solution to the household's problem, including the multipliers, Λ_{1t}^i and Λ_{2t}^i.

A.1.2 Goods Producing Firms

The problem faced by a typical goods producer is given in the text in Eq. (5). The associated first-order conditions are:

$$P_t F_k(K_t, N_t; z_t) = R_t$$
$$P_t F_n(K_t, N_t; z_t) = W_t$$

A.1.3 Government

In addition to the expressions for lump-sum transfers, Eq. (6), the stock of money evolves according to

$$M_{t+1} = \mu_t M_t.$$

A.1.4 Aggregates

Total capital, labor, money and consumption are given, respectively, by:

$$K_t = \sum_{i=0}^{T-1} k_{t-i}^i,\ N_t = \sum_{i=0}^{T-1} n_{t-i}^i,\ M_{t+1} = \sum_{i=0}^{T-1} M_{t-i}^{i+1},\ \text{and } C_t = \sum_{i=0}^{T-1} c_{t-i}^i.$$

A.1.5 Conversion to Real Magnitudes

Normalize nominal variables by the aggregate price level:

$$w_{t+i} \equiv \frac{W_{t+i}}{P_{t+i}}, \quad r_{t+i} \equiv \frac{R_{t+i}}{P_{t+i}}, \quad x_{t+i} \equiv \frac{X_{t+i}}{P_{t+i}},$$
$$m_t^{i+1} \equiv \frac{M_t^{i+1}}{P_{t+i}}, \quad \lambda_{1t}^i \equiv \Lambda_{1t}^i P_{t+i}, \quad \lambda_{2t}^i \equiv \Lambda_{2t}^i P_{t+i}, \quad \pi_{t+i} \equiv \frac{P_{t+i}}{P_{t+i-1}}, \quad m_{t+1} \equiv \frac{M_{t+1}}{P_t}.$$

Notice that money balances are normalized by the 'previous period' price level; this is done so that the household's budget constraint does not involve next period's price level which is not known at the time household decisions are made.

The equations governing the solution of this economy are:

$$c_t^i + k_t^{i+1} + m_t^{i+1} = w_{t+i} n_t^i + [1 + \delta + r_{t+i}]\, k_t^i + \frac{m_t^i}{\pi_{t+i}} + x_{t+i} \quad i = 0, \ldots, T-1$$

$$c_t^i = \frac{m_t^i}{\pi_{t+i}} + x_{t+i}, \quad i = 0, \ldots, T-1$$

$$U_c(c_t^i, 1-n_t^i) = \left[\lambda_{1t}^i + \lambda_{2t}^i\right], \quad i = 0, \ldots, T-1$$

$$U_\ell(c_t^i, 1-n_t^i) = \lambda_{1t}^i w_{t+i}, \quad i = 0, \ldots, T-1$$

$$\lambda_{1t}^i = \beta E_{t+i}\left\{\lambda_{1t}^{i+1}\left[1-\delta + r_{t+i+1}\right]\right\}, \quad i = 0, \ldots, T-2$$

$$\lambda_{1t}^i = \beta E_{t+i}\left\{\frac{\lambda_{1t}^{i+1} + \lambda_{2t}^{i+1}}{\pi_{t+i+1}}\right\}, \quad i = 0, \ldots, T-2$$

$$k_t^0 = 0, \quad k_t^t = 0$$

$$m_t^0 = \overline{m}, \quad m_t^t = \overline{m}$$

$$r_t = F_k(K_t, N_t; z_t)$$

$$w_t = F_n(K_t, N_t; z_t)$$

$$K_t = \sum_{i=0}^{T-1} k_{t-i}^i$$

$$N_t = \sum_{i=0}^{T-1} n_{t-i}^i$$

$$m_{t+1} = \sum_{i=0}^{T-1} m_{t-i}^{i+1}$$

$$m_{t+1} = \mu_t \frac{m_t}{\pi_t}$$

$$x_t = \frac{(\mu_t - 1)m_t/\pi_t}{T}$$

A.2 Non-monetary Model

A.2.1 Household's Problem

The household's Bellman equation is:

$$V(k_t^i; i) \equiv \max \Bigg\{ U(c_t^i, 1-n_t^i) + \beta E_{t+i} V(k_t^{i+1}; i+1)$$
$$+ \lambda_t^i \big[w_{t+i} n_t^i + [r_{t+i} + 1 - \delta] k_t^i - (c_t^i + k_t^{i+1})\big] \Bigg\}$$

The Euler equations and budget constraint are:

$$c_t^i + k_t^{i+1} = w_{t+i} n_t^i + [r_{t+i} + 1 - \delta] k_t^i, \quad i = 0, \dots, T-1$$

$$U_2(c_t^i, 1 - n_t^i) = \lambda_t^i w_{t+i}, \quad i = 0, \dots, T-1$$

$$U_1(c_t^i, 1 - n_t^i) = \lambda_t^i, \quad i = 0, \dots, T-1$$

$$\lambda_t^i = \beta E_{t+i} \lambda_t^{i+1} [r_{t+i+1} + 1 - \delta], \quad i = 0, \dots, T-2$$

$$k_t^0 = 0, \quad k_t^t = 0$$

A.3 The Pareto Problem

The Lagrangian for the planner's problem is

$$\mathcal{L} = \sum_{t=-T+1}^{0} \omega_t \left[\sum_{i=T+t}^{T-1} \beta^i U(c_t^i, 1 - n_t^i) \right] + \sum_{t=1}^{\infty} \omega_t \left[\sum_{i=0}^{T-1} \beta^i U(c_t^i, 1 - n_t^i) \right]$$
$$+ \sum_{t=0}^{\infty} \lambda_t \left[F\left(K_t, \sum_{i=0}^{T-1} n_{t-i}^i \right) + (1 - \delta) K_t - \sum_{i=0}^{T-1} c_{t-i}^i - K_{t+1} \right]$$

The first-order conditions are:

$$c_t^i: \qquad \omega_t \beta^i U_c(c_t^i, 1 - n_t^i) - \lambda_{t+i} = 0 \tag{A.1}$$

$$n_t^i: \qquad \omega_t \beta^i U_\ell(c_t^i, 1 - n_t^i) - \lambda_{t+i} F_n(K_t, N_t) = 0 \tag{A.2}$$

$$K_{t+1}: \qquad -\lambda_t + \lambda_{t+1} \left[F_k(K_{t+1}, N_{t+1}) + 1 - \delta \right] = 0 \tag{A.3}$$

where $N_t = \sum_{i=0}^{T-1} n_{t-i}^i$.

Eqs. (A.1) and (A.2) can be combined to give

$$U_\ell(c_t^i, 1 - n_t^i) = U_c(c_t^i, 1 - n_t^i) F_n(K_t, N_t). \tag{A.4}$$

Eqs. (A.1) and (A.3) yield

$$\omega_t \beta^i U_c(c_t^i, 1 - n_t^i) = \omega_{t+1} \beta^i U_c(c_{t+1}^i, 1 - n_{t+1}^i) \left[F_k(K_{t+1}, N_{t+1}) + 1 - \delta \right]. \tag{A.5}$$

Assume that the weight attached to generation t is given by $\omega_t = \beta^t$; in other words, the planner discounts utility of each generation by the same discount factor used by households. Eq. (A.1) now reads

$$\beta^{t+i}U_c(c_t^i, 1-n_t^i) - \lambda_{t+i} = 0 \tag{A.6}$$

Now, consider an agent born at $t-1$, aged $i+1$:

$$\beta^{t+i}U_c(c_{t-1}^{i+1}, 1-n_{t-1}^{i+1}) - \lambda_{t+i} = 0 \tag{A.7}$$

Eqs. (A.6) and (A.7) imply

$$U_c(c_t^i, 1-n_t^i) = U_c(c_{t-1}^{i+1}, 1-n_{t-1}^{i+1}). \tag{A.8}$$

Focus on steady state in which $c_t^i = c^i$, $n_t^i = n^i$ and $K_t = K$. Eqs. (A.4), (A.5) and (A.8) now read

$$U_\ell(c^i, 1-n^i) = U_c(c^i, 1-n^i)F_n(K, N) \tag{A.9}$$

$$1 = \beta\left[F_k(K, N) + 1 - \delta\right] \tag{A.10}$$

$$U_c(c^i, 1-n^i) = U_c(c^{i+1}, 1-n^{i+1}). \tag{A.11}$$

A.4 Computational Issues

Notice that the aggregate state vector includes the capital and money holdings of all cohorts alive at a particular date. Individual decision rules depend on the entire state vector (not merely a few selected moments) since the state vector is needed to form expectations of future prices (which, in turn, depend on the future state vector). Fortunately, as pointed out by Ríos-Rull (1996), matters are greatly simplified if decision rules are *linear*. When solving for decision rules when the stochastic elements of the model are in play, it is then opportune to use a log linearization technique. See Klein (2000) for details on the particular technique employed in this paper.

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