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The Cyclicality of Search Intensity in a Competitive Search Model

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Worker Search Effort as an Amplification Mechanism*

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Abstract

Shimer's puzzle is that the textbook Diamond-Mortensen-Pissarides model exhibits fluctuations in labor market variables that are an order of magnitude too small. Introducing search effort of the unemployed brings the model's predictions for these fluctuations very close to those seen in the data. The search cost function and the matching technology are intimately related and thus should be estimated simultaneously. Ignoring worker search effort leads to a large upward bias in the elasticity of matches with respect to vacancies. Evidence in support of the model's prediction of procyclical search effort is presented.

Keywords: Variable Search Effort, Unemployment and Vacancies, Endogenous Matching Technology, Time Use, Wage Posting, Competitive Search, Unemployment Volatility by Education

JEL Codes: E24, E32, J63, J64

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1 Introduction

The Diamond-Mortensen-Pissarides (DMP) model of search and matching is a widely accepted model of equilibrium unemployment. Shimer (2005) argues that the textbook version of the DMP model underpredicts, by an order of magnitude, the cyclical variability in key labor market variables that are central to this theory, namely vacancies and unemployment; similar results are also found in Andolfatto (1996) and Merz (1995). In this paper, we introduce worker search effort as in Pissarides (2000, Ch. 5). As a result, workers can take direct action to affect the outcome of their labor market search, a channel absent from most previous studies of the DMP model, an exception being Merz (1995). We show that search effort by the unemployed serves as a strong amplification mechanism in the DMP model.

We make an innocuous change to the DMP framework, dropping what Rogerson, Shimer and Wright (2005) refer to as the black box of the Nash bargaining solution determination of wages in favor of competitive search which entails wage posting by firms and directed search on the part of the unemployed; see Moen (1997) and Rogerson *et al.* (2005).[1] Wage posting is motivated by two facts. First, as documented by Hall and Krueger (2012), wages of newly-hired workers with less than college education are predominantly determined through wage posting, not bargaining. Second, working with data from the Current Population Survey (CPS) reveals that over 85% of the cyclical variation in unemployment is due to individuals with less than college education; see Figure 1.

[Figure 1 about here.]

Workers' search cost is central to our paper, and its calibration is an important contribution of our quantitative analysis. We discipline our calibration of the search cost by targeting micro evidence on the elasticity of search intensity with respect to unemployment insurance benefits estimated by Krueger and Mueller (2010). They estimate the elasticity of time spent on job search with respect to unemployment benefits by using cross-state variation in

[1] As pointed out by Veracierto (2009), it is unclear what value should be used for the Nash bargaining parameter, nor is it clear how this parameter might change over the business cycle.

unemployment insurance benefits, benefit eligibility and dependent allowances while controlling for a rich set of worker characteristics.[2] The idea behind this aspect of our calibration strategy is simple: In the search and matching framework, the impact of a marginal increase in benefits is equivalent to the impact of a marginal decrease in productivity. Empirically, this calibration strategy allows us to circumvent problems associated with using aggregate time series, namely the paucity of micro-level observations at each point in time, and the fact that the data covers less than one full business cycle; see Section 2 for details.

The benchmark calibration explains over 80% of the standard deviation of unemployment and the vacancies-unemployment ratio, and 90% of that of vacancies. Below, we show analytically that in the presence of endogenous worker search effort, labor market volatility is mainly determined by gross flow income while unemployed. Hagedorn and Manovskii (2008) also show that flow income while unemployed is an important determinant of labor market volatility. They show that the DMP model can successfully explain the cyclical volatility of vacancies and unemployment when the surplus of a match is around 5% (the flow value of being unemployed is roughly 95%) of labor productivity. In Hagedorn and Manovskii, flow income while unemployed is very close to that of the employed: being employed raises a worker's flow income by 2.3%. Mortensen and Nagypál (2007) question whether workers work in order to increase their income by roughly 2% over what they would receive while unemployed. In contrast to Hagedorn and Manovskii, in our benchmark calibration, the employed enjoy 12% higher income than the unemployed. So, one can generate sufficient volatility in unemployment and vacancies by using a high gross flow income for the unemployed while still maintaining a substantial employment surplus through the low net utility for the unemployed.

To evaluate the role of search effort in our model, we also solve the model with fixed

[2] Similarly to Krueger and Mueller (2010), Rothstein (2011) uses differences in the unemployment insurance policy across the U.S. states to quantify the impact of the recent sharp expansions of unemployment insurance benefits on job search. More recently, Hagedorn, Karahan, Manovskii and Mitman (2013) exploit discontinuity of unemployment insurance policies at state borders (i.e., by comparing counties that border with each order but belong to different states) and emphasize the importance of the responsiveness of job creation to unemployment benefit extensions (also see the discussion at the end of Section 6.2).

search intensity. In this case, the model accounts for about 15% of the standard deviation of unemployment, almost 40% of the variability in the vacancies-unemployment ratio, and nearly 60% of the volatility in vacancies. That is to say, endogenous search effort is an important ingredient of the model, with its effects working most strongly through unemployment, and so the vacancies-unemployment ratio.

To understand the role of search effort in our model, first consider the model without an effort dimension. As described in Shimer, an increase in productivity increases the value of a match. As a consequence, firms post more vacancies which boosts the workers' job finding rate, raising workers' outside option (the value of being unemployed). The net result is that wages rise, eating up much of the gain associated with the increase in productivity, thereby lowering the response of vacancies. *With effort*, the productivity increase leads the unemployed to search more intensively which dampens the rise in the value of being unemployed, and so the increase in the wage. In this case, the smaller increase in the wage leaves more of the surplus for firms, thus amplifying the response of vacancies. There is a sort of virtuous circle in which the increase in vacancies leads workers to search more which leads to more vacancies, and so on.

The results in this paper would be vacuous if we were unconstrained in our choice of the search cost function. Section 5 shows analytically that the properties of this cost function are constrained by the elasticity of the matching function with respect to the vacancy-unemployment ratio. Empirical plausibility then places strong restrictions on the search cost. While our analytical results point to the importance of variable search intensity in our model, a highly elastic search intensity would likely be inconsistent with the data on unemployment and vacancies, and particularly the elasticity of matches with respect to the vacancies-unemployment ratio.

Yashiv (2000) appears to be the only paper that estimates the matching technology when search intensity of the unemployed is endogenous; he used Israeli data.[3] In general, ignoring search intensity may be an important oversight. The results in Section 7 show that neglecting search intensity introduces a large upward bias in the elasticity of the number of matches with respect to vacancies; this result is consistent with the empirical work of Yashiv. For the benchmark calibration, omitting search effort would lead one to erroneously conclude that a 10% increase in vacancies would increase the number of matches by more than 5% whereas the actual impact is less than 2%. Such a discrepancy should make one cautious in interpreting results from equilibrium search and matching models with fixed search intensity, particularly when quantitatively evaluating the effects of alternative public policies such as the effects of unemployment benefits, employment subsidies, and job search assistance.

Another, even more important implication of the findings in Section 7 concerns the Nash bargaining parameter, which is central to standard search and matching theory. In the literature, the Nash bargaining parameter is usually inferred from data on unemployment and vacancies (Shimer, 2005; Mortensen and Nagypál, 2007). Specifically, guided by the Hosios (1990) condition, a worker's bargaining power is set to the the elasticity of matching function with respect to unemployment. The results in Section 7 suggest that the common method of estimating bargaining power exhibits a strong downward bias. For example, the numerical results show that when the elasticity of matching with respect to unemployment is 0.456, the worker's bargaining power parameter required to achieve the constrained efficient allocation is not 0.456, but rather 0.802. Conversely, picking the bargaining parameter based on the measured elasticity of the matching function with respect unemployment or vacancies cannot always guarantee constrained efficiency. These results point to one of the benefits of adopting competitive search instead of Nash bargaining determination of wages. For the standard DMP model, the allocations associated with competitive search are always efficient;

[3] Yashiv's (2000) principal contributions are to estimate the various frictions in the matching process, including the matching function, firm search, and worker search. He does not perform a quantitative evaluation of the model like that contained herein, nor does he provide analytical results as we do. Christensen, Lentz, Mortensen, Neumann and Werwatz (2005) and Lise (2013) also estimate search cost functions, but co-mingle search by the unemployed with on-the-job search; neither do they jointly estimate the search cost and matching functions.

see Moen (1997).

The outline of the rest of the paper is as follows. Section 2 surveys the literature on the cyclical properties of search effort as well as presenting some evidence on its cyclicality. Section 3 presents a dynamic, stochastic model of equilibrium unemployment incorporating variable search intensity into a competitive search model. Section 4 presents key analytical results characterizing the equilibrium. Section 5 explores the steady-state properties of the model. The steady state analysis is important for calibrating a number of key parameters in the model. The model is calibrated and simulated in Section 6, establishing the model's business cycle properties. To evaluate the impact of endogenous search on the business cycle properties of labor market variables, we present experiments in which search effort is constant. Implications of variable search intensity on the aggregate matching technology are discussed in Section 7. Section 8 concludes. The appendices provide further analytical details.

2 The cyclical properties of search effort

A key prediction from search models with endogenous search effort is that effort is procyclical. Any empirical evidence regarding search effort of the unemployed is, by its nature, indirect since we do not directly observe the amount of effort expended by a searcher. Introspection provides little help since one can construct a case for both procyclical and countercyclical search intensity. Search effort will be countercyclical if, during recessions, the unemployed are motivated to search more intensively in the face of an otherwise falling job-finding rate. Alternatively, recessions are lousy times to be looking for a job; since the returns are low, search effort "should be" procyclical. Do bears spend a lot of time and effort searching for honey during the winter when there is little honey to be found, or during the summer when it is plentiful? Is there any reason to think that the unemployed in the U.S. are not smarter than the average bear? In the remainder of this section, we present an indirect measure of search intensity, look at some interstate mobility data that sheds some light on search effort, as well as review the available evidence presented elsewhere in the literature.

2.1 Average search intensity

Our measure of search intensity is obtained from the aggregate matching function, in much the same way that a measure of aggregate productivity can be obtained by performing a "Solow residual exercise." As is common in the literature, assume that the matching function is Cobb-Douglas:

$$m_t = v_t^{\eta} \left(s_t u_t \right)^{1-\eta}, \quad 0 < \eta < 1 \tag{1}$$

where m_t is matches (equivalently, new hires) at time t, v_t is vacancies posted by firms, u_t is the level of unemployment, and finally s_t is average search effort. In writing the matching function as in equation (1), all changes in matches that cannot be attributed to variation in vacancies or unemployment are attributed to changes in average search intensity. Vacancies are measured by the Conference Board's Help-Wanted Index, extended past the mid-1990s by Barnichon (2010). Unemployment is given by total civilian unemployment from the Bureau of Labor Statistics. Following Shimer (2005), matches are obtained from

$$u_{t+1} = u_t - m_t + u_t^{\mathrm{s}}$$

where u_t^{s} is short-term unemployment (less than five weeks). This equation says that the number of people unemployed next month is equal to the number unemployed this month, less those who found jobs (matches), plus those who lost jobs (short-term unemployed).[4] The final task is to assign a value to η, the elasticity of matches with respect to vacancies. As discussed in our calibration section, estimates of η in the literature are biased when search effort is endogenous. Using our calibrated value, $\eta = 0.215$, Figure 2 presents imputed

[4] As is common in the search unemployment literature, this equation ignores flows in and out of the labor force.

average search intensity.[5] While this series is noisy – perhaps owing to the fact that the underlying data are monthly – it is clear that average search effort falls sharply during NBER recessions (the shaded regions in the figures). In two of the more recent recessions, average search intensity has continued to fall after the "official" end of the recession. Overall, our imputed average search effort series clearly exhibits a countercyclical pattern, falling during recessions and rising gradually during expansions.

[Figure 2 about here.]

We also present another series of imputed search intensity while considering the empirical Beveridge curve proposed by Mortensen and Nagypál (2007): $\hat{f}_t = \frac{\lambda(1-u_t)}{u_t}$, where $\hat{f}_t$ is the job-finding rate and λ is the separation rate. Then, using equation (1),

$$s_t = \hat{f}_t^{\frac{1}{1-\eta}} \left(\frac{v_t}{u_t}\right)^{\frac{\eta}{\eta-1}}. \tag{2}$$

As before, using our calibrated values, $\eta = 0.215$ and $\lambda = 0.0083$, Figure 2 presents imputed average search intensity. In this case, the pattern is even clearer: measured search intensity drops sharply during every recession, then gradually increases during expansions.

2.2 Evidence from labor mobility

Labor mobility provides novel insight to job search behavior.[6] For this analysis, we use the March CPS (1980–2012) of adult civilians aged 16–64 who are in the labor force. Table 1 shows that job-related moves are the dominant reason for interstate moves, and so we focus on such long distance moves. Figure 3 plots interstate mobility along with the unemployment rate, after removing their respective linear time trends. These two series are strongly

[5]The data presented in Figure 2 is unfiltered. We obtain quite similar results Hodrick-Prescott filtering the data with a smoothing parameter of 10^5 as in Shimer (2005).

[6]The role of labor mobility for the aggregate labor market has been long recognized. For example, Friedman (1968) suggests that labor mobility might be a key determinant of aggregate unemployment. Lucas and Prescott (1974) and Rogerson (1987) analyze the impact of sectoral mobility on the aggregate labor supply.

negatively correlated (with a correlation coefficient of -0.610 at the significance level 0.003), showing that mobility is strongly procyclical.[7] Of course, it is possible that individuals make long distance moves only after they have a job in hand. To investigate this possibility, we look at how unemployment differs between movers and stayers. The idea is that if individuals tend to move across state borders once they have found a job, then unemployment among recent movers should be lower than among stayers. In our sample, the unemployment rate among recent movers is 9.2% while it is 5.2% among stayers. This observation suggests that individuals move in order to improve their prospects of finding a job, not because they have already found a job.

[Table 1 about here.]

[Figure 3 about here.]

Of course, one could argue that there are systematic differences between movers and stayers, like age and education. To account for this possibility, we run the following regression:

$$h_{i,j,t} = \delta D_{i,t} + \phi X_{i,j,t} + e_{i,j,t} \tag{3}$$

where $h_{i,j,t}$ is a dummy for whether person i who is living in state j in year t is unemployed, $D_{i,t}$ is a dummy for whether the person is a mover, $X_{i,j,t}$ is a set of controls such as state and year effects, age and education, and $e_{i,j,t}$ is the error term. Table 2 shows that among observationally equivalent workers, movers are still more likely to be unemployed than stayers. In interpreting this result, it should be kept in mind that a typical local market experiences simultaneous in- and out-migration and these two flows are much larger than the corresponding net migration, meaning that for an average move in the U.S., the unemployment rates

[7] Recent work by Moscarini and Vella (2008) and Kambourov and Manovskii (2009) shows that industrial and occupational mobility are also procyclical. While this may also suggest procyclical search effort, in the data, it is hard to isolate the individual level relationship between unemployment and occupational/industrial mobility. In other words, these other studies cannot determine the timing of mobility and unemployment, nor can they tell whether individuals switching occupations and industries are more likely to be unemployed. This is why we focus here on geographical mobility.

at the mover's origin and destination do not differ much (see, for example, Lkhagvasuren, 2012).

[Table 2 about here.]

It is likely that the CPS understates the gap in the unemployment rate between movers and stayers. In particular, the CPS reports an individual's mobility status in March. However, it is well known that geographic mobility peaks in the summer. Consequently, someone who moves over the summer without a job in hand has six to nine months over which they could find a job before the March survey. Therefore, the unemployment gap between movers and stayers, δ, can be much higher than measured in Table 2. So, the mobility data indicate that this form of job search effort is strongly procyclical.

2.3 More direct measures of search intensity

Shimer (2004) is an early and influential work trying to infer the cyclical properties of search effort of the non-employed. Using data from the CPS, Shimer uses the number of search methods as a proxy for search effort; he finds that search effort is countercyclical. Tumen (2012) has revisited Shimer's results. He points out that the CPS measures the number of search methods used during a respondant's most recent unemployment spell. It turns out that the number of search methods used increases with the length of an unemployment spell. Since the duration of unemployment rises during recessions, it follows that the number of search methods used will also rise during recessions. Tumen proposes using the number of search methods used divided by the length of the unemployment spell as a proxy for search effort. He argues that this measure corresponds to the arrival rate of new search methods within an unemployment spell. Tumen finds that his measure of search intensity is procyclical.

Krueger and Mueller (2010) also provide indirect evidence that search effort is procyclical. Using data on time spent on job search from the American Time Use Survey (ATUS) as a measure of search effort, they find that time spent on search increases with a worker's expected wage. While the aggregate wage is only mildly procyclical, Solon, Barsky and Parker (1994) show that individual wages are strongly procyclical, the difference being due to a composition bias. Since recessions are times during which workers have lower expected wages, the Krueger and Mueller evidence suggests that time spent on job search by the unemployed is likely procyclical.

Data from the American Time Use Survey (ATUS) has led to analysis of the actual time spent on job search by the unemployed; see Krueger and Mueller (2010), DeLoach and Kurt (forthcoming) and Aguiar, Hurst and Karabarbounis (2013). In Figure 4, we present average time spent on search by the unemployed based on our own calculations using data from the ATUS. The dates of the Great Recession as given by the NBER are indicated by the light gray shaded area. In addition to average time spent on search, 95% confidence bands are included, and the data is sliced not only annually but also quarterly and monthly.[8] Start with the annual data. In 2008, the first full year of the Great Recession, average search time rose to 47.1 minutes from 33.5 minutes in 2007. However, the standard errors are large (20.2 minutes for 2008 and 13.4 minutes for 2007). Based on a 95% confidence interval, one cannot say for certain that average time spent on job search by the unemployment changed between 2007 and 2008. Higher frequency data results in more volatility in the average search time series, and even larger confidence intervals. Even if more careful analysis of the ATUS reveals that time spent by the unemployed on job search is countercyclical, this data currently covers but one recession.

[Figure 4 about here.]

There is a sizable micro-labor literature on the responses of the unemployed to the policy parameters of unemployment insurance programs; important early contributions include Katz and Meyer (1990) and Meyer (1990). Some common findings in this literature are:

[8]Standard errors are computed using the methodology laid out in the *American Time Use Survey User's Guide*.

holding fixed the number of weeks of unemployment, the probability of exiting unemployment falls with the replacement rate (the unemployment insurance benefit divided by the previous wage), and the exit rate from unemployment rises sharply around the time that an unemployed individual exhausts his/her benefits. These empirical regularities are taken as *prima facie* evidence that the unemployed adjust their search effort in response to these unemployment insurance program policy parameters. This interpretation of the evidence is typically justified with reference to a search model with endogenous search intensity. Using this evidence to make inferences about the cyclicality of search effort involves a couple of steps. To start, in this model, an increase in unemployment benefits has the *same effect* as a fall in the wage. The next link in the chain of reasoning is to again note that individual wages are highly procyclical. Therefore, the micro-labor evidence on the effects of changes in unemployment insurance benefits provides indirect evidence that search effort is procyclical.

2.4 Summary

To summarize the empirical evidence concerning the cyclicality of the search intensity of the unemployed, we presented two related measures of average search effort, both of which are clearly procyclical. Interstate mobility, which we consider as an essential component of job search effort, is procyclical. While Shimer (2005) finds that search effort, proxied by the number of search methods used, is countercyclical, Tumen (2012), using search methods used per week, finds that search intensity is procyclical. At the aggregate level, ATUS data on time spent on job search by the unemployed is not sufficiently precisely measured to definitively say anything about the cyclicality of time spent on job search, and this data only covers one recession. Krueger and Mueller's (2010) finding that time spent searching is positively related to a worker's expected wage provides indirect evidence that search effort is procyclical.[9] As argued above, the interpretation of micro-labor estimates of the effects of

[9] Marinescu (2012) shows that the number of weeks of unemployment benefits has a negative impact on the number of job applications at the state level; this result is consistent with Krueger and Mueller (2010) and Tumen (2012).

changes in the unemployment benefit on worker search equally apply to the effects of changes in the real wage, and imply that worker search effort is procyclical.

3 Model

3.1 Environment

The economy is populated by a measure one of infinitely-lived, risk-neutral workers and a continuum of infinitely-lived firms. Individuals are either employed or unemployed.[10] An unemployed worker looks for a job by exerting variable search effort. The cost of searching for a job depends on how intensively the worker searches. Let $s_i \geq 0$ be the search intensity of worker i. The cost of s_i units of search is $c(s_i)$ where c is a twice continuously differentiable, strictly increasing and strictly convex function. Flow utility of unemployed worker i is given by $z - c(s_i)$ where $z \geq 0$. Normalize the cost of search so that $c(0) = 0$, implying that z is flow utility of an unemployed worker who exerts zero search intensity. Flow utility of an employed worker is the wage, w. Workers and firms discount their future by the same factor $0 < \beta < 1$.

A firm employs at most one worker. Per-period output of a firm-worker match is denoted by p and evolves according to a stationary and monotone Markov transition function $G(p'|p)$ given by

$$p' = 1 - \varrho + \varrho p + \sigma\varepsilon, \tag{4}$$

where ε is an *iid* standard normal shock, $0 < \varrho < 1$ and $\sigma > 0$. As in Shimer (2005), mean productivity is normalized to 1. There is free entry for firms. A firm finds its employee by posting a vacancy, at the per period cost k, when looking for workers. All matches are dissolved at an exogenous rate λ. Matches are formed at random; the matching technology is discussed shortly.

[10] Shimer (2004) suggests that labor market participation reflects search effort. We follow the usual practice in the literature in abstracting from flows in and out of the labor force. Consequently, our model is silent on the participation margin.

3.2 Wage determination

Wages are determined via competitive search instead of Nash bargaining. The setup follows Rogerson *et al.* (2005). Given current productivity, p, a firm decides whether or not to post a vacancy. If it does, the firm decides what wage to offer in order to maximize its expected profits. An unemployed worker directs her search towards the most attractive job given current aggregate labor market conditions. Let $\tilde{w}$ denote the expected present discounted value of a wage stream offered by a vacant job. A vacant job is fully characterized by $(p, \tilde{w})$. Let $\mathcal{W}(p)$ denote the set of present discounted values associated with wage streams posted in the economy when aggregate productivity is p.

3.3 Matching technology

Matching between firms and workers operates as follows. Let $s_{i,j}$ denote search effort by unemployed worker i for job type $j = (p, \tilde{w})$ where it is understood that $s_{i,j}$ can be non-zero for at most one j. (There is no on-the-job search.) Since a worker searches for at most one type of job, $s_i = \max_j\{s_{i,j}\}$. Let $\mathbf{U}_j$ denote the set of u_i unemployed workers searching for a type j job. The total search intensity exerted by these workers is $S_j = \displaystyle\int_{\mathbf{U}_j} s_{i,j} di$. Denote total vacancies of type j by v_j. As in Pissarides (2000, Ch. 5), the total number of matches formed for a particular job type is given by the Cobb-Douglas function,

$$M_j = \mu {v_j}^{\eta} {S_j}^{1-\eta} \tag{5}$$

where $0 < \eta < 1$ and $\mu > 0$. The (effective) queue length for a type j vacant job is given by $q_j = S_j/v_j$, and the probability that a particular job is filled is given by $\alpha(q_j) = \mu q_j^{1-\eta}$. The probability that an unemployed worker i finds a job of type j is $f(q_j)s_{i,j}$ where $f(q_j) = \mu/q_j^{\eta}$. Let θ_j denote labor market tightness for a type j job: $\theta_j = v_j/u_j$. For notational brevity, the individual index i is omitted for the rest of the paper.

3.4 Value functions

Workers

Let $W(\tilde{w}, p)$ denote the value to a worker of a new job offering $\tilde{w}$ when the current state is p. Let $U(p)$ denote the value of being unemployed. Then, the value of searching for a job offering $\tilde{w}$ when aggregate productivity is p is given by

$$\begin{aligned}\tilde{U}(\tilde{w}, p) \equiv \max_{s_{\tilde{w},p}} \Big\{ & z - c(s_{\tilde{w},p}) + \beta f(q_{\tilde{w},p}) s_{\tilde{w},p} \int W(\tilde{w}, p') dG(p'|p) \\ & + \beta \left(1 - f(q_{\tilde{w},p}) s_{\tilde{w},p}\right) \int U(p') dG(p'|p) \Big\}. \end{aligned} \tag{6}$$

Since an unemployed worker chooses to search for the job that yields the highest expected utility,

$$U(p) \equiv \max_{\tilde{w} \in \mathcal{W}(p)} \{\tilde{U}(\tilde{w}, p)\}, \tag{7}$$

where we anticipate the result that there are a finite number of elements in $\mathcal{W}(p)$.

The value of a new job consists of two main components: the expected present value of the wage stream and the expected value of unemployment upon future separation. Specifically, $W(\tilde{w}, p)$ is given by

$$W(\tilde{w}, p) = \tilde{w} + \int Q(p') dG(p'|p) \tag{8}$$

where Q is determined by the following equation: $Q(p) = \beta\lambda U(p) + \beta(1-\lambda) \int Q(p') dG(p'|p)$.

Firms

Let $Z(p)$ denote the value of the expected output streams of a firm when the current state is p: $Z(p) = p + \beta(1 - \lambda) \int Z(p') dG(p'|p)$. Then, the value of a new match to a firm offering $\tilde{w}$ to its employee is given by:

$$J(\tilde{w}, p) = \int Z(p') dG(p'|p) - \tilde{w}. \tag{9}$$

Finally, the value of a vacancy when productivity is p is given by

$$V(p) = \max_{\tilde{w}} \left\{-k + \beta\alpha(q_{\tilde{w},p})J(\tilde{w}, p)\right\}. \tag{10}$$

The formal definition of the labor market equilibrium is provided in Appendix A. The remainder of the paper establishes the main properties of the equilibrium.

4 Equilibrium characterization

Since unemployed workers are intrinsically identical and direct their search to the most attractive jobs, the value of unemployment $U(p)$ is common across all workers (also see Rogerson *et al.*, 2005). Consequently, the non-wage component of the value of employment, $Q(p)$, is also common across jobs. It is important to keep these in mind for our analysis below.

A worker will take the queue length, $q_{\tilde{w},p}$, as given. The first-order condition with respect to search intensity, $s_{\tilde{w},p}$, in equation (6) is

$$c'(s_{\tilde{w},p}) = \beta f(q_{\tilde{w},p}) \left[W^e(\tilde{w}, p) - U^e(p)\right], \tag{11}$$

where $U^e(p) = \displaystyle\int U(p')dG(p'|p)$ and $W^e(\tilde{w}, p) = \displaystyle\int W(\tilde{w}, p')dG(p'|p)$. As in Rogerson *et al.* (2005), firms making their wage posting decision will take equation (11) as given. Specifically, a firm's problem in equation (10) can be reduced to the following:

$$\max_{q_{\tilde{w},p}} \alpha(q_{\tilde{w},p})J(\tilde{w}, p) \tag{12}$$

subject to equation (11). Then, since $\frac{dJ(\tilde{w},p)}{d\tilde{w}} = -\frac{dW^e(\tilde{w},p)}{d\tilde{w}} = -1$, the first order condition for the firm's problem can be rewritten as

$$\alpha'(q_{\tilde{w},p})J(\tilde{w},p) - \alpha(q_{\tilde{w},p})\frac{dW^e(\tilde{w},p)}{dq_{\tilde{w},p}} = 0. \tag{13}$$

Then, inserting equation (11) for $W^e(\tilde{w},p)$ into equation (13), we obtain

$$\alpha'(q_{\tilde{w},p})J(\tilde{w},p) = \frac{c'(s_{\tilde{w},p})}{\beta}\eta. \tag{14}$$

Next, the free entry condition implies that $J(\tilde{w},p) = k/(\beta\alpha(q_{\tilde{w},p}))$. Therefore, equation (13) becomes

$$c'(s_{\tilde{w},p})q_{\tilde{w},p} = \frac{k(1-\eta)}{\eta}. \tag{15}$$

Proposition 1 (**Same jobs**)**.** *Given current productivity, all firms creating a vacancy offer the same level of the present discounted value of wages.*

Proof. See Appendix A.2 □

Proposition 1, along with the free entry condition, implies that the vacancies created within the same period have the same queue length, that is $q_{\tilde{w},p}$ is unique to productivity p. Then, using equation (15), one can make the following claim:

Corollary 1 (**Same search intensity**)**.** *All unemployed workers exert the same search intensity.*

It should be made clear that these results are obtained without making any specific assumption on the shape of the wage profile for a given match.[11] Given the uniqueness result, we drop the subscripts of s, q and θ. Then, equation (15) can be rewritten as

$$qc'(s) = \frac{k(1-\eta)}{\eta} \tag{16}$$

[11] We are very grateful to an anonymous referee for directing us toward this equilibrium characterization, which uses transferability of utility between a firm and its employee. In a previous version of the paper, we obtained equation (16) and the uniqueness results (Proposition 1 and Corollary 1) by imposing a constant wage within a match.

or, equivalently,

$$sc'(s) = \frac{k(1-\eta)}{\eta}\theta. \tag{17}$$

Equations (16) and (17) represent one of our key analytical results. Specifically, it shows that in equilibrium, labor market tightness, θ, and search intensity, s, are positively related.

For expositional purposes, the rest of the analysis proceeds in two steps. Section 5 analyzes the steady state of the model. In particular, the model is solved analytically for constant productivity and key parametric relationships that are important to the calibration of the model are obtained. In Section 6, productivity is once again stochastic, and the main properties of the model are established numerically.

5 Steady state analysis

Here we consider the case in which productivity, p, is constant over time. Proceeding as in the previous section, it can be shown (see Appendix A.3) that in equilibrium,

$$p - z = \frac{1-\beta(1-\lambda)}{\beta\alpha'(q)}c'(s) + c'(s)s - c(s). \tag{18}$$

5.1 Permanent productivity shock

Now, consider the response of the model economy to a permanent shift in productivity.

Proposition 2 (Permanent shock). *An increase in productivity raises both search intensity and the vacancy-unemployment ratio, and therefore raises the job-finding rate.*

Proof. Given the inverse relationship between queue length, q, and worker search intensity, s, the right hand side of equation (18) is strictly increasing in s. Therefore, s increases with productivity, p. A higher s and a lower q means a higher vacancy-unemployment ratio. More vacancies per unemployed worker along with higher search intensity imply a higher job-finding rate. □

Given the strict convexity of the search cost function, c, equation (17) implies that market tightness, θ, is strictly increasing with search intensity, s. More importantly, in light of Proposition 2, equation (17) suggests that the volatility of the vacancy-unemployment ratio is closely related to the search cost. This relation is quantified in the following section.

5.2 The elasticity of the vacancy-unemployment ratio to productivity

Next we extend the analytical results in Hagedorn and Manovskii (2008) and Mortensen and Nagypál (2007) to the model with variable search intensity. Specifically, we calculate the elasticity of the vacancy-unemployment ratio to productivity, defined as $\frac{d\ln\theta}{d\ln p}$, and compare it with that in a standard model or, equivalently, the model with fixed search intensity.

Let $\tilde{\eta}$ denote the implied (or empirical) elasticity of the job-finding rate with respect to the vacancy-unemployment ratio, that is, $\tilde{\eta} = \frac{d\ln(f(q)s)}{d\ln\theta}$. Without loss of generality, normalize the initial search intensity to 1. Then, by taking logs in equation (18) and differentiating the result with respect to $\ln p$, it can be shown that (see Appendix A.5)

$$\frac{d\ln\theta}{d\ln p} = \frac{p}{p-z} \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})} + \left(1 - \frac{c(1)}{c'(1)}\right)\left(1 + \frac{c'(1)}{c''(1)}\right)}{\frac{1-\beta(1-\lambda)}{\beta f(q)} + 1}. \tag{19}$$

Given convexity of the search cost function it follows that $0 < \frac{c(1)}{c'(1)} < 1$ and $\frac{c'(1)}{c''(1)} > 0$, and therefore, $C \equiv \left(1 - \frac{c(1)}{c'(1)}\right)\left(1 + \frac{c'(1)}{c''(1)}\right) > 0$. In steady state, the unemployment rate is $\frac{\lambda}{\lambda+f(q)}$. Given that the average unemployment rate for the U.S. is around 6% (Shimer, 2005), it follows that $\frac{\lambda}{\lambda+f(q)} \simeq 0.06$ which implies $f(q) \gg \lambda$. When the model period is relatively short, the discount factor, β, is close to 1 and so $\frac{1-\beta(1-\lambda)}{\beta f(q)} \simeq \frac{\lambda}{f(q)}$ is much smaller than 1. Further, the observed elasticity $\tilde{\eta} \simeq 0.5$ (Petrongolo and Pissarides, 2001; Mortensen and Nagypál, 2007) and so $\frac{1-\beta(1-\lambda)}{\beta f(q)}\frac{1}{1-\tilde{\eta}} \simeq \frac{\lambda}{f(q)}\frac{1}{1-\tilde{\eta}}$ is also much smaller than 1. The upshot is that the magnitude of the elasticity $\frac{d\ln\theta}{d\ln p}$ is dictated by $\frac{p}{p-z}$ and $\left(1 - \frac{c(1)}{c'(1)}\right)\left(1 + \frac{c'(1)}{c''(1)}\right)$.

Clearly, the magnitude of this elasticity can be made arbitrarily large by assuming a cost function such that $\frac{c(1)}{c'(1)} \ll 1$ and $\frac{c'(1)}{c''(1)} \gg 1$. However, doing so will lead to highly counterfactual implications for the matching technology. Specifically, using the fact that $\frac{d\ln\alpha(q)}{d\ln q} \leq 1$,

$$C = \left(1 - \frac{c(1)}{c'(1)}\right)\left(1 + \frac{c'(1)}{c''(1)}\right) < 1 + \frac{c'(1)}{c''(1)} = \frac{1}{1-\tilde{\eta}}\frac{d\ln\alpha(q)}{d\ln q} \leq \frac{1}{1-\tilde{\eta}} \simeq 2.193, \tag{20}$$

where the value $\tilde{\eta} = 0.544$ is obtained from Mortensen and Nagypál (2007). So, the empirical elasticity of the matching function, $\tilde{\eta}$, dictates that C can not be much larger than 2. In fact, if search costs are given by a power function – a commonly-used specification (e.g., Christensen *et al.*, 2005; Nakajima, 2012; and Lise, 2013) – then the value of C is much lower than 2. Specifically, let the function c be given by the following power function:

$$c(s) = \chi s^{\gamma}, \tag{21}$$

where $\chi > 0$ and $\gamma > 1$. Under such a parametric specification, $C = 1$, regardless of the values of χ and γ.

With the cost function in equation (21), equation (19) becomes

$$\frac{d\ln\theta}{d\ln p} = \frac{p}{p-z} \times \underbrace{\frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})} + 1}{\frac{1-\beta(1-\lambda)}{\beta f(q)} + 1}}_{K}. \tag{22}$$

For comparison purposes, we also calculate the above elasticity for the model with fixed search intensity. Here the model with fixed search intensity refers to the model where search intensity is fixed to 1 while the elasticity of the matching function and the unemployment rate are matched with their empirical counterparts. Under fixed search intensity, the elasticity is given by (see Appendix B.3 for derivation)

$$\frac{d\ln\theta^{\mathrm{F}}}{d\ln p} = \frac{p}{p-(z-c(1))} \times \underbrace{\frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})}+1}{\frac{1-\beta(1-\lambda)}{\beta f(q)}+1}}_{K}. \tag{23}$$

Given our calibration in Section 6, $\frac{p}{p-z} = 12.690$, $\frac{p}{p-(z-c(1))} = 7.622$ and $K = 1.071$. These numbers imply that $\frac{d\ln\theta}{d\ln p} = 13.596$ while $\frac{d\ln\theta^{\mathrm{F}}}{d\ln p} = 8.166$. So, the elasticity in the two models is determined by either z relative to productivity p (in the case of equation (22)) or $z-c(1)$ relative to p. Using our calibrated values, search effort amplifies the elasticity of the vacancy-unemployment ratio with respect to a permanent change in productivity by almost 70%, specifically, $\frac{d\ln\theta}{d\ln p}/\frac{d\ln\theta^{\mathrm{F}}}{d\ln p} = 1.665$.

What is more surprising is that, despite the introduction of search intensity, the elasticity given by equation (22) coincides with that obtained by Hagedorn and Manovskii (2008) and Mortensen and Nagypál (2007) for the textbook version of the DMP model after imposing the Hosios condition and given the flow income of unemployment, z. Given this result, we make the following two key observations:

(a) As in the standard model, the elasticity of vacancy-unemployment ratio with respect to productivity in the model with variable search effort is determined by $\frac{p}{p-z}$, which is consistent with Hagedorn and Manovskii (2008).

(b) However, an important difference is that the net flow utility of an unemployed worker in the model with variable search intensity is $z-c(1)$ while that in the standard model (that is, the one without variable search intensity) it is simply z. Consequently, the employment surplus can be substantially higher in the model with variable search effort.

In summary, one can generate a sufficient volatility in unemployment and vacancies by using a high gross flow income for the unemployed (that is, high z) while still maintaining a substantial employment surplus through the low net utility for the unemployed, $z-c(s)$.

Given our cost function, C in equation (20) is 1. Lower values of z, the flow income while unemployed, then imply a lower elasticity of the vacancy-unemployment ratio with respect to productivity. In this case, we could get a higher value for this elasticity by choosing a non-power cost function that brings C closer to its upper bound of around 2.2. For example, from equation (22), to match the benchmark elasticity of 13.596 when C is at its upper bound would allow us to lower z to 0.839. We choose not to follow this route, following instead Christensen *et al.* (2005), Nakajima (2012) and Lise (2013) in using a power function, equation (21). In fact, our numerical analysis in Section 6 shows that this cost function performs well for the model in the sense that the volatility of vacancies and unemployment are remarkably close to their empirical counterparts, despite the fact that these moments were not targeted during the calibration.

5.3 Main intuition

In the remainder of the section, we discuss the basic intuition behind the amplifying effect of the variable search cost. There are three main equilibrium channels that are key to understanding the amplifying effect of the variable search cost. The first effect arises from the complimentarity of search intensity, reflected in the equilibrium condition in equation (17). When there is an increase in productivity p, firms create more vacancies and workers search more intensely. The nature of the complimentarity is that as firms increase vacancies, workers search even more, leading firms to post more vacancies, and so on. The second main effect operates through the interaction of search cost and profits. Specifically, an increase in worker search effort lowers the flow utility of unemployment. As a result, the match surplus remains relatively large and firm profits are large enough to encourage a large increase in vacancies (see Appendices A.6 and B.4). The final effect is a shift in the Beveridge curve arising from the effect of search intensity on the workers' arrival rate of job offers.

How do these effects translate into the equilibrium level of unemployment and vacancies?

To answer this question, combine equations (16) and (18) to obtain

$$p - z = \frac{1 - \beta(1-\lambda)}{\beta\mu} \left(\frac{k}{\eta}\right)^{\eta} \left(\frac{\chi\gamma}{1-\eta}\right)^{1-\eta} s^{(\gamma-1)(1-\eta)} + \chi(\gamma-1)s^{\gamma}. \tag{24}$$

Equation (24) shows that search intensity, s, is an increasing function of productivity, p. Combining this result with equation (17), the vacancy-unemployment ratio, θ, is an increasing function of p. Using a diagram similar to those of Pissarides (2000, Chapter 5), the impact of productivity on the vacancy-unemployment ratio is depicted as a counterclockwise rotation of the job creation (JC) curve in the vacancy-unemployment plane in Figure 5. The standard model with fixed effort also exhibits a rotation of the JC curve, but not as large as with endogenous search effort (see Appendix B.4).

[Figure 5 about here.]

On the other hand, the Beveridge curve (BC) is given by

$$u = \frac{\lambda}{\lambda + \mu\left(\frac{v}{u}\right)^{\eta}(s)^{1-\eta}}. \tag{25}$$

It is easy to notice that, due to the positive response of search intensity to an increase in productivity, the Beveridge curve shifts left (see Figure 5). The intersection of the two curves gives the equilibrium level of unemployment and vacancies. The shift in the Beveridge curve, along with the increase in labor market tightness, imply that search effort amplifies the effects of a productivity change on unemployment, and has an ambiguous effect on vacancies. Our numerical results below show that search effort amplifies the volatility of vacancies as well. This means that under a reasonable calibration, the effect of the shift in the Beveridge curve on vacancies is dominated by the shift in the job creation curve. In summary, adding worker search effort amplifies the responses of labor market tightness, vacancies and the unemployment rate to permanent changes in productivity.

6 Business cycle properties

Here, productivity is stochastic and the model is solved numerically.

6.1 Calibration

Standard parameters

The length of the time period is a quarter of a month, which will be referred to as a week. The discount factor β is set to $1/1.04^{1/48}$, a value consistent with an annual real interest rate of 4%. The separation rate is set to that in Shimer (2005); normalizing it to a weekly frequency, $\lambda = \frac{0.1}{12} = 0.0083$. The productivity process $G(p'|p)$ is approximated by a five-state Markov chain using the method of Rouwenhorst (1995).[12] The following targets for the productivity process are taken from Hagedorn and Manovskii (2008): the quarterly autocorrelation of 0.765, and the unconditional standard deviation of 0.013 for the HP-filtered productivity process with a smoothing parameter of 1600. At a weekly frequency, these targets require setting: $\varrho = 0.9903$ and $\sigma = 0.0033$.

Normalization

Following Shimer (2005), the target for the mean vacancy-unemployment ratio is 1. Then, it follows that the effective queue length is $q = 1$ in steady state and therefore, recalling that productivity, p, has been normalized to 1, equations (17) and (18) can be rewritten as

$$z = 1 - \frac{(1-\beta(1-\lambda))\chi\gamma}{\beta(1-\eta)\mu} - \chi(\gamma - 1) \tag{26}$$

and

$$k = \chi\gamma\frac{\eta}{1-\eta}. \tag{27}$$

[12] Galindev and Lkhagvasuren (2010) show that for highly persistent autoregressive processes, the method of Rouwenhorst (1995) outperforms other commonly-used discretization methods.

Given the rest of the parameter values, z and k are chosen according to equations (26) and (27). The value of μ, the scaling parameter in the matching function, is chosen by targeting an average unemployment rate of 5.7% (Shimer, 2005).

The elasticity of matches to vacancies

The key parameter of the matching technology is the elasticity of matches with respect to vacancies, $\epsilon_{M,v} = \frac{\partial \ln M}{\partial \ln v}$. When search intensity is fixed, this elasticity is given by η (Petrongolo and Pissarides, 2001). However, when search intensity is allowed to vary, the measured elasticity of matches to vacancies, $\epsilon_{M,v}$, differs from η. Specifically, combining equation (16) with equations (21) and (27) gives $s^\gamma = \theta$. On the other hand, given the uniqueness result in Proposition 1, total search intensity is simply $S = us$ where u denotes unemployment. These results imply that, under variable search intensity, the equilibrium number of matches is given by

$$M = \mu v^{1-(1-\eta)\left(1-\frac{1}{\gamma}\right)} u^{(1-\eta)\left(1-\frac{1}{\gamma}\right)}. \tag{28}$$

At this point, there are two important conclusions:

(a) The property that the matching function is constant returns to scale with respect to unemployment and vacancies is preserved under variable search intensity. This result is consistent with the fact that empirical studies do not reject constant returns to scale in the matching functions; see the survey of Petrongolo and Pissarides (2001).

(b) Under endogenous job search effort, the implied elasticity of matches with respect to vacancies is given by

$$\epsilon_{M,v} = 1 - (1-\eta)\left(1 - \frac{1}{\gamma}\right). \tag{29}$$

Given the value of γ, η is chosen such that $\epsilon_{M,v} = 0.544$, an elasticity estimate obtained by Mortensen and Nagypál (2007).

The search cost

The scaling parameter, χ, is calibrated such that the average disutility per minute of work is equal to the average disutility per minute of job search. Given that average search intensity is normalized to one, the average flow cost of job search is approximately χ. The average disutility of job search is, then, χ/T_u where T_u is time spent on job search. Recall that z represents the per-period flow of utility of an unemployed worker who exerts zero search effort. This flow utility consists of unemployment benefits received during unemployment, b, and the imputed value of leisure, ℓ:

$$z = b + \ell. \tag{30}$$

b is set to 0.3 as in Mortensen and Nagypál (2007). The disutility of work is, then, ℓ and the disutility per unit time is ℓ/T_w where T_w is time spent working. Consequently, the parameter χ is set such that

$$\frac{\chi}{T_u} = \frac{\ell}{T_w}. \tag{31}$$

According to the 2008 ATUS, a typical unemployed worker spends 40.47 minutes per weekday on job search activities. The same survey reveals that employed workers spend, on average, 39.99 hours per week working. These numbers imply that the ratio of search time to work time is $T_u/T_w = 0.0844$.[13]

The only remaining parameter is γ which governs the curvature of the workers' search cost function. This parameter is important to the analysis since the responsiveness of time spent on job search to aggregate productivity depends heavily on the value of γ. For example, if γ is very high, there will be relatively little variation in search intensity and the model will behave similarly to those in previous studies. Equation (24) provides useful insight into how to quantify the responsiveness of search intensity to labor market conditions. This equation

[13] This calibration of χ may be conservative since it focuses exclusively on time spent on job search (relative to time spent working). For example, the unemployed may suffer a psychic utility loss associated with being rejected for a job.

can be rewritten as

$$p - (b + \ell) = \frac{1 - \beta(1 - \lambda)}{\beta\mu} \left(\frac{k}{\eta}\right)^{\eta} \left(\frac{\chi\gamma}{1 - \eta}\right)^{1-\eta} s^{(\gamma-1)(1-\eta)} + \chi(\gamma - 1)s^{\gamma}, \tag{32}$$

where, as above, ℓ is the imputed value of leisure. Given equation (32), a marginal increase in unemployment benefits, b, and a marginal decrease in productivity, p, have the same effect on search intensity, s. This important feature of the model is used to calibrate the search cost. The curvature parameter, γ, is pinned down by calibrating to the elasticity of job search with respect to unemployment benefits, denoted $\epsilon_{s,b}$. Using cross-state differences, Krueger and Mueller (2010) find that time spent on job search is inversely related to the generosity of unemployment benefits. They estimate that the elasticity of time spent on job search with respect to unemployment benefits is between -2.235 and -1.579. The calibration target for $\epsilon_{s,b}$ is -1.907, a value halfway between the two estimates.[14] This elasticity is computed in the model as follows. The stochastic model is solved for the benchmark value of unemployment benefits, b. The model is, then, re-solved for a value of b that is 1% higher than its benchmark value. The elasticity is then calculated as the percentage change in (average) search time divided by the percentage change in unemployment benefits.

The value of γ, 2.3858, means that worker search costs are roughly quadratic. This finding is consistent with the estimates of Yashiv (2000) using Israeli data, and with Christensen *et al.* (2005) who used micro data from Denmark on wages and employment. Lise (2013) estimates an elasticity substantially lower than quadratic, albeit for white males in the U.S., and including on-the-job search (as did Christensen *et al.*). Nakajima (2012) calibrated roughly a quadratic search cost function in order for his model to be consistent with empirical evidence concerning the response of the average duration of unemployment to changes in the unemployment insurance policy parameters.

[14] Among workers with lower annual income, Krueger and Mueller (2010) find that the elasticity of search intensity is -2.7. This value implies that search intensity is more elastic among lower educated workers, who have much higher cyclical unemployment than their more educated counterparts. This result is reassuring in that Figure 1 shows that the bulk of U.S. unemployment variability is due to workers with low educational attainment.

6.2 Results

[Table 3 about here.]

For the remainder of the paper, the current calibration will be referred to as the benchmark model. Table 3 shows that the model matches the calibration targets. Table 4 displays the parameters of the benchmark model. The average search cost, $\mathbb{E}(\chi s^{\gamma})$, measured relative to labor productivity, is 0.05. Average flow utility while unemployed is approximately 0.869, expressed relative to labor productivity, implying that the employment surplus is approximately 12 percent of labor productivity.

[Table 4 about here.]

Table 5 presents summary statistics of quarterly U.S. data of 1951Q1–2011Q4 and predictions of the benchmark model. The results show that the benchmark model accounts for 80% of the observed volatility of both the vacancy-unemployment ratio and unemployment, and 90% of the percentage standard deviation of vacancies. Search intensity is pro-cyclical with a standard deviation of 9 percent.[15] In Section 2.1, we inferred average search intensity from data on vacancies and unemployment. We presented two measures of average search; after taking logarithms and Hodrick-Prescott filtering, their standard deviations are 5.1 percent ("Shimer" measure) and 10.6 percent ("Mortensen-Nagypal" method). By this metric, our model's prediction for the standard deviation of search effort seems reasonable. The model performs reasonably well along other dimensions.

[Table 5 about here.]

Hagedorn and Manovskii (2008) estimate that the elasticity of wages with respect to productivity is 0.449 and target this elasticity in their calibration. In the benchmark model, the elasticity of wages of new matches with respect to productivity is $\epsilon_{w,p} = 0.472$. This elasticity is comparable to the one targeted by Hagedorn and Manovskii (2008), although it should

[15] In the spirit of the business cycle literature which defines the cycle as deviations of output from trend, here the cycle is defined as deviations of labor productivity from trend.

be noted that the elasticity of the wages of new matches is calculated under the assumption that the wage of a particular match does not change over time. In Appendix A.7, this assumption is relaxed; depending on how frequently wages of old matches are re-negotiated, the elasticity of the average wage with respect to productivity ranges between 0.041 (if the wages of old matches are not re-negotiated at all) and 0.967 (if the wages of old matches are re-negotiated following each aggregate shock). Therefore, it makes little sense to target the elasticity of the wage with respect to productivity in a model with wage posting. At the same time, it also means that one can target any level of the elasticity in the above range by introducing an additional parameter for the frequency of wage renegotiation.

The role of the flow utility while unemployed is also explored in Table 5; see the "Lower z" case. Specifically, z is set to 0.75 which leaves the average flow utility while unemployed at 70% of average productivity. In this case, the model accounts for roughly a quarter of the observed volatility in vacancies, unemployment, and their ratio. However, the discussion at the end of Section 5.2 suggests that model's predictions for the volatility of labor market variables could be enhanced by choosing an alternative search cost function that comes closer to setting the parameter C in equation (22) to its upper bound.

As a further test of the model, we evaluate its prediction for the effect of an increase in unemployment insurance benefits on the duration of unemployment. There is a large micro-labor literature estimating this effect. The bulk of the evidence says that a ten percent increase in benefits increases the average duration of unemployment spells by 0.5 to 1.5 weeks; (see, for example, Meyer, 1990). Our benchmark model predicts that, in response to a ten percent increase in benefits, the average duration of unemployment increases by roughly 2.1 weeks – somewhat higher than the range cited above. One possible explanation for this discrepancy is that in the model, unlike Nakajima (2012), we have abstracted from the possibility of an unemployed person exhausting his/her benefits; the micro-labor literature reveals that the job-finding rate rises sharply around the time that benefits are exhausted. Another, perhaps more important, reason for this discrepancy is that, as Hagedorn *et al.*

(2013) point out, micro studies on the impact of benefits ignore the equilibrium effect of job creation by firms and thus underestimate the impact of benefits. Nonetheless, the model predictions for this effect are reasonable, even though we did not target this moment.

6.3 The net impact of variable search intensity

How much of the success of the benchmark model can be attributed to variable search intensity? To answer this question, the model is solved while fixing search intensity. The problems of workers and firms in the stochastic model with fixed search intensity is provided in Appendix B. In the absence of shocks to productivity, the model with fixed search intensity is identical to the competitive search model of Rogerson *et al.* (2005).

Two cases are considered. First, the model is solved while fixing search intensity at one and using the same parameter value in the matching function, η, as in the benchmark economy. Table 5 shows that fixing search intensity reduces the percentage standard deviation of the vacancy-unemployment ratio by more than a half. While the percentage standard deviation of vacancies falls by around a third, that of unemployment is reduced by 80%, leaving its volatility at just under 15% of that seen in the data (compared to over 80% for the benchmark model).

These results show that approximately 45% ($\simeq \frac{0.216-0.1}{0.259}$) of the observed volatility of the vacancy-unemployment ratio is explained by variable search effort. Repeating the same calculation for unemployment, search intensity explains roughly 67% ($\simeq \frac{0.103-0.019}{0.125}$) of the volatility of cyclical unemployment. By this same metric, search intensity accounts for about 32% ($\simeq \frac{0.126-0.082}{0.139}$) of the volatility of vacancies. In other words, search intensity has a much larger impact on the percentage standard deviation of unemployment than vacancies.

Under this specification, the implied elasticity of matches with respect to vacancies is $\epsilon_{M,v} = \eta = 0.1984$, which is much lower than its empirical counterpart 0.544. To target the latter under fixed search intensity, the model is simulated while setting η to 0.544 and keeping search intensity at one. In this case, fixed search effort leads to a smaller decline in unemployment volatility and a larger decline in that of vacancies. However, the volatility of the vacancy-unemployment ratio is almost the same as for the first fixed effort experiment.

7 Implications on the matching technology

In this section, we analyze the implications of search intensity on labor-market matching.

7.1 Interdependence of matching and search intensity

When search intensity is fixed, the elasticity of the number of matches with respect to vacancies, $\epsilon_{M,v}$, coincides with the matching technology parameter η: $\epsilon_{M,v} = \eta$. However, under endogenous job search effort, the elasticity is given by $\epsilon_{M,v} = 1 - (1-\eta)\left(1 - \frac{1}{\gamma}\right)$ (see Section 6.1). Consequently, the parameter η can differ substantially from $\epsilon_{M,v}$, the elasticity measured directly from data on cyclical unemployment, vacancies and matches. For example, for the benchmark calibration, $\eta = 0.215$ and $\epsilon_{M,v} = 0.544$. Thus, if one ignores variable search intensity, one would erroneously conclude that a one percent exogenous increase in vacancies will raise the number of matches by more than 0.5 percent whereas the actual impact could be less than 0.2 percent.

These results show that the matching technology and the costs of search are intimately related. To estimate the two functions simultaneously requires an equilibrium framework that allows for endogenous search effort. This paper offers one such a framework.

7.2 Shifts in the Beveridge curve

Throughout this paper, labor market fluctuations have been modeled as arising due to productivity shocks. However, Mortensen and Nagypál (2007) point out that the correlation between labor productivity and the vacancy-unemployment ratio is less than unity and emphasize the importance of other omitted driving forces. Consistent with their argument, a sizable fraction of the variation of matches is not explained by shifts in unemployment and vacancies (Petrongolo and Pissarides, 2001). In this context, variation of matches means overall shifts in the number of matches, which includes both cyclical fluctuations and the trend.

The results in this paper suggest that part of these unexplained shifts in matches (or, equivalently, changes in the parameters of the matching function) can be attributed to shifts in the costs associated with vacancy creation and job search. For instance, equation (32) shows that increases in the cost parameters k, χ and γ, reduce equilibrium search intensity. Therefore, in general, the total number of matches is given by

$$M(k, \chi, \gamma, v, u) = A(k, \chi, \gamma) v^{\eta} u^{1-\eta}, \tag{33}$$

where A is a decreasing function of its arguments. As a result, the number of matches for a given level of unemployment and vacancies can shift with these cost parameters. Equation (33) has the following two important implications.

First, Lubik (2011) and Furlanetto and Groshenny (2013) argue that a negative shock to match efficiency A (or, a mismatch shock) is consistent with the outward shift of the U.S. Beveridge curve in the aftermath of the Great Recession. This finding, along with equation (33), raises the possibility that an increase in the above cost parameters might be key to understanding persistently high unemployment despite an increased number of vacancies during the recent recovery.

Second, cross-country data show that there are substantial differences in unemployment across countries. Empirical studies have tended to focus on whether taxes or benefits can explain these cross-country unemployment differences; see, for example, Prescott (2004) and Ljungqvist and Sargent (2006). Time spent on job search also differs substantially across countries. For example, according to Krueger and Mueller (2010), an average unemployed worker spends 41 minutes a day searching for a job in the U.S., compared with just 12 minutes in the average European country. The results in this paper suggest that differences in time spent on job search, or equivalently, differences in search or vacancy creation costs, may account for a substantial part of the cross-country differences in unemployment.

8 Conclusion

We modify the textbook DMP model by adding worker search intensity, allowing workers to directly affect the outcome of their job search over the business cycle. We also introduce a far more innocuous change, the dropping Nash bargaining determination of wages in favor of competitive search (a combination of wage posting and directed search). Calibration of the search cost function is disciplined by data from the American Time Use Survey, including some innovative work by Krueger and Mueller (2010) that provides evidence concerning the elasticity of search with respect to unemployment benefits.

The benchmark model predicts standard deviations for unemployment and the vacancies-unemployment ratio that are 80% of that seen in the U.S. data; for vacancies, the model captures nearly 90% of the observed volatility. Almost all of the improvement with respect to the variability of unemployment – and about half of that for the vacancies-unemployment ratio – can be traced to the introduction of endogenous worker search effort. The volatility of vacancies is not much affected by worker search intensity. We show that one can generate a sufficient volatility in unemployment and vacancies by using a high gross flow income for the unemployed while still maintaining a substantial employment surplus through low net utility for the unemployed. While more elastic search effort can improve the model's performance, our analytical results show that there are limits to this channel. Specifically, a highly elastic search effort would likely be inconsistent with the data on unemployment and vacancies, and particularly the elasticity of matches with respect to the vacancies-unemployment ratio.

To date, endogenous worker search effort has been largely overlooked when estimating the matching technology; a notable exception is Yashiv (2000). Section 7 shows that this omission can lead to an overestimate, by a factor of 2.5, of the effects on job matching of an increase in vacancies. This problem is not merely of academic interest since it has implications for public policies aimed at reducing unemployment. The results also suggest that when wages are determined by Nash bargaining, choosing the bargaining power of workers based on estimates of matching functions alone is premature and cannot always guarantee constrained efficiency.

The model could also be used to analyze shifts in the Beveridge curve and cross-country differences. For example, the analytical results in this paper raise the possibility that an increase in job search and job creation costs might have lead the observed outward shift of the Beveridge curve in the aftermath of the Great Recession. Moreover, Krueger and Mueller (2010) report that time spent on job search is much higher in the U.S. than in Europe. On the other hand, it is well known that unemployment is substantially lower in the U.S. than in Europe. Since the model establishes a negative correlation between time spent on job search and unemployment in an equilibrium framework, it may be interesting to use the model to examine whether cross-country differences in worker search costs can account for the cross-country differences in unemployment.

A Model with variable search intensity

A.1 The definition of the labor market equilibrium

Since unemployed workers are intrinsically identical, it follows that $U(p)$ is common to all unemployed workers. Further, $\tilde{U}(\tilde{w}, p)$ must be the same for all jobs for which workers actually search. It then follows that the queue length, $q_{\tilde{w},p}$, must be unique for all jobs with positive worker search: The compensation for searching for a lower wage job is a higher probability of being matched, that is, a lower queue length. Using equations (6) and (7), it can be seen that search intensity, $s_{\tilde{w},p}$, must also be unique for each job type $(\tilde{w}, p)$. Introducing the following functions, $s(\tilde{w}, p) = s_{\tilde{w},p}$, $q(\tilde{w}, p) = q_{\tilde{w},p}$, $v(\tilde{w}, p) = v_{\tilde{w},p}$, $u(\tilde{w}, p) = u_{\tilde{w},p}$ and $S(\tilde{w}, p) = S_{\tilde{w},p}$ for any $(p, \tilde{w})$ such that $\tilde{w} \in \mathcal{W}(p)$, the labor market equilibrium can now be defined.

Definition A.1. *The equilibrium is a set of value functions, $\{U, W, J, V\}$, a decision rule s, a set of the present discounted value of the wages, $\mathcal{W}$, the measures, $\{u, v\}$, the total search intensity, S, and the queue length, q, such that*

1. *unemployed: given q and W, the decision rule $s(\tilde{w}, p)$ and the value functions $U(p)$ and $\tilde{U}(\tilde{w}, p)$ solve equations (6) and (7) for any $\tilde{w} \in \mathcal{W}(p)$;*
2. *employed: given U, the value function $W(\tilde{w}, p)$ solves equation (8);*
3. *matched firm: the value function $J(\tilde{w}, p)$ solves equation (9);*
4. *vacancy: given q and J, the wage $\tilde{w}$ and value function $V(p)$ solve equation (10) with $\tilde{w} \in \mathcal{W}(p)$;*
5. *free entry: for any real number x,*

$$\begin{cases} v(x,p) > 0 \text{ and } V(p) = 0 & \text{if } x \in \mathcal{W}(p), \\ v(x,p) = 0 \text{ and } V(p) \le 0 & \text{if } x \notin \mathcal{W}(p) \text{ or } \mathcal{W}(p) = \emptyset; \text{ and} \end{cases} \tag{A.1}$$

6. *consistency: the total search intensity S and the queue length q are consistent with*

individuals' and firms' behavior: $S(\tilde{w},p) = u(\tilde{w},p)s(\tilde{w},p) = v(\tilde{w},p)q(\tilde{w},p)$ *for* $\tilde{w} \in \mathcal{W}(p)$.

A.2 Proof of Proposition 1

Let $Z^e(p) = \int Z(p')dG(p'|p)$ and $R(p) = \int \left(\int Q(p'')dG(p''|p') \right) dG(p'|p)$. Then, equation (11) can be rewritten as

$$\frac{c'(s_{\tilde{w},p})}{\beta f(q_{\tilde{w},p})} = \tilde{w} + R(p) - U^e(p). \tag{A.2}$$

On the other hand, using the free entry condition,

$$\frac{k}{\beta \alpha(q_{\tilde{w},p})} = -\tilde{w} + Z^e(p). \tag{A.3}$$

Combining equations (A.2) and (A.3), it can be seen that

$$\frac{c'(s_{\tilde{w},p})}{\beta f(q_{\tilde{w},p})} + \frac{k}{\beta \alpha(q_{\tilde{w},p})} = Z^e(p) + R(p) - U^e(p).$$

Furthermore, using equation (16),

$$\frac{k}{\beta \eta \alpha(q_{\tilde{w},p})} = Z^e(p) + R(p) - U^e(p).$$

The right hand side of the equation is common across all jobs posted at a given point in time. Since α is a strictly increasing function, $q_{\tilde{w},p}$ is unique across vacancies. Then, the free entry condition in equation (A.3) implies that $\tilde{w}$ is the same across all vacancies posted at a given point in time.

A.3 The steady state characterization

When there are no shocks to productivity, i.e. when p is constant over time, a job is fully characterized by its per-period wage $w = (1 - \beta(1 - \lambda))\tilde{w}$. The value of being unemployed is given by

$$U = \max_s \{z - c(s) + \beta f(q) s \big(W - U\big) + \beta U\} \tag{A.4}$$

and the value of being employed is

$$W = \frac{w + \beta\lambda U}{1 - \beta(1 - \lambda)}. \tag{A.5}$$

A worker will take the queue length, q, as given. Differentiating the right hand side of equation (A.4) with respect to search effort, s, gives

$$c'(s) = \beta f(q)(W - U).$$

Combining this result with equations (A.4) and (A.5), it can be shown that the optimal search intensity must satisfy the following:

$$w - z = \frac{1 - \beta(1 - \lambda)}{\beta f(q)} c'(s) + c'(s)s - c(s). \tag{A.6}$$

Firms making their vacancy posting decision will take equation (A.6) as given. The value of a vacancy can be written as

$$V = \max_w \{-k + \beta\alpha(q)\frac{p - w}{1 - \beta(1 - \lambda)}\}. \tag{A.7}$$

Following Rogerson *et al.* (2005), substitute equation (A.6) into equation (A.7) for w and thereby reduce a firm's problem to the following:

$$\max_q \left\{ \alpha(q) \left(p - z - \frac{1 - \beta(1 - \lambda)}{\beta f(q)} c'(s) - c'(s)s + c(s) \right) \right\}. \tag{A.8}$$

Taking the FOC with respect to q yields equation (18).

A.4 Normalizations

Suppose that the search intensity is normalized to $x > 0$. Let the associated search cost function be $\tilde{c}$. Denote the vacancy cost and the coefficient of the matching function by $\tilde{k}$ and $\tilde{\mu}$, respectively. The equilibrium allocations continue to be characterized by equations (17) and (18). Then, it can be seen that the same allocation is obtained by choosing the cost function to satisfy $\tilde{c}'(x)x - \tilde{c}(x) = c'(1) - c(1) > 0$ while setting $\tilde{k} = \frac{x\tilde{c}'(x)}{c'(1)}k$ and $\tilde{\mu} = \frac{x^{\eta}\tilde{c}'(x)}{c'(1)}\mu$.

As in Shimer (2005), the normalization of θ, the vacancy-unemployment ratio, is inconsequential to the results. Consider another value, say $\overline{\theta}$, for the mean vacancy-unemployment ratio. Then, it can be seen that multiplying k and μ by $\overline{\theta}$ and $\overline{\theta}^{\eta}$, respectively, leaves the equilibrium allocations given by equations (17) and (18) unaffected.

A.5 Productivity and the vacancy-unemployment ratio

The implied elasticity of the job-finding rate with respect to the vacancy-unemployment ratio can be written as

$$\tilde{\eta} = \frac{d\ln(f(q)s)}{d\ln\theta} = \frac{d\ln(q\alpha(q)s)}{d\ln\theta} = \frac{d\ln(\theta\alpha(q))}{d\ln\theta} = 1 + \frac{d\ln\alpha(q)}{d\ln\theta}. \tag{A.9}$$

Since $\ln\theta = \ln s - \ln q$, equation (A.9) can be written as

$$\tilde{\eta} - 1 = \frac{\epsilon_{q,s}}{1-\epsilon_{q,s}}\frac{d\ln\alpha(q)}{d\ln q}, \tag{A.10}$$

where $\epsilon_{q,s} = \frac{d\ln q}{d\ln s}$. Recalling that $\theta = s/q$, differentiation of equation (16) gives $\epsilon_{q,s} = -\frac{sc''(s)}{c'(s)}$ in equilibrium. Differentiate $\ln\theta = \ln s - \ln q$ with respect to $\ln p$ to obtain the elasticity of the vacancy-unemployment ratio θ with respect to productivity p:

$$\frac{d\ln\theta}{d\ln p} = (1-\epsilon_{q,s})\frac{d\ln s}{d\ln p}, \tag{A.11}$$

As normalized in Section 5.2, let $s = 1$. Then, by taking logs in equation (18) and differentiating the result with respect to $\ln p$, it can be shown that

$$\frac{d\ln s}{d\ln p} = \frac{p}{p-z} \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})}\frac{c'(1)}{c''(1)+c'(1)} + \frac{c'(1)-c(1)}{c''(1)}}{\frac{1-\beta(1-\lambda)}{\beta f(q)}+1}. \tag{A.12}$$

Now combining equations (A.11) and (A.12) along with $\epsilon_{q,s} = -\frac{c''(1)}{c'(1)}$, one can arrive at

$$\frac{d\ln\theta}{d\ln p} = \frac{p}{p-z} \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})} + \left(1-\frac{c(1)}{c'(1)}\right)\left(1+\frac{c'(1)}{c''(1)}\right)}{\frac{1-\beta(1-\lambda)}{\beta f(q)}+1}. \tag{A.13}$$

A.6 Elasticity of the profit with respect to productivity

Combining the free entry condition $k = \beta\alpha(q)\frac{p-w}{1-\beta(1-\lambda)}$ with equations (A.12) and (A.13), the elasticity of a firm's profit with respect to productivity is given by

$$\frac{d\ln(p-w)}{d\ln p} = \frac{p}{p-z} \times (1-\tilde{\eta}) \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})} + \left(1-\frac{c(1)}{c'(1)}\right)\left(1+\frac{c'(1)}{c''(1)}\right)}{\frac{1-\beta(1-\lambda)}{\beta f(q)}+1}. \tag{A.14}$$

When $c(s) = \chi s^{\gamma}$, this equation is further simplified to

$$\frac{d\ln(p-w)}{d\ln p} = \frac{p}{p-z} \times \underbrace{\frac{\frac{1-\beta(1-\lambda)}{\beta f(q)}+1-\tilde{\eta}}{\frac{1-\beta(1-\lambda)}{\beta f(q)}+1}}_{L}. \tag{A.15}$$

Comparing this result with equation (B.16), the profit is more sensitive to productivity in the model with variable search intensity than that in the model with fixed search intensity. Specifically, using our calibrated values, it can be seen that the the elasticity is 70% higher in the model with variable search intensity. So, the wage moves less in the model with fixed search intensity due the the effects mentioned in Section 5.3.

A.7 Volatility of the wage

Given productivity p, Proposition 1 states that the expected present discounted value of the wage stream of new matches is unique. Let $\tilde{w}(p)$ denote this unique value. Then, under the assumption that the wage of a particular match does not change over time, the wage of new matches is given by

$$w(p) = (1 - \beta(1 - \lambda))\tilde{w}(p). \tag{A.16}$$

For the benchmark calibration, the elasticity of $w(p)$ with respect to productivity is $\epsilon_{w,p} = 0.472$; this value is comparable to the one targeted by Hagedorn and Manovskii (2008). Since most of the matches are old, the elasticity of the *average* wage in the model economy must be much smaller. In fact, under the assumption of constant within-match wages, the elasticity of the *average* wage with respect to productivity is 0.041.

Suppose instead that the wages of old matches are allowed to evolve over time. Specifically, let the wages of all matches, new or old, be the same and respond to each aggregate shock. Let $\hat{w}$ be the wage determined in such a way. Then, it can be seen that

$$\tilde{w}(p) = \hat{w}(p) + \beta(1 - \lambda)\mathbb{E}_p\tilde{w}(p'). \tag{A.17}$$

Combining the latter with equation (A.16) gives

$$\hat{w}(p) = \frac{1}{1 - \beta(1 - \lambda)} \left[w(p) - \beta(1 - \lambda)\mathbb{E}_p w(p') \right]. \tag{A.18}$$

In this case, the elasticity of $\hat{w}(p)$ with respect to productivity is 0.967. Therefore, for the benchmark model, depending on how frequently wages of old matches are renegotiated after each aggregate shock, the elasticity of the average wage with respect to productivity ranges from 0.041 to 0.967.

B Model with fixed search intensity

B.1 Workers

When search intensity is fixed at one, the flow utility of unemployment becomes

$$\tilde{z} = z - c(1).$$

Then, the value of being unemployed is given by

$$U(p) = \tilde{z} + \beta f(q) \left[\mathbb{E}_p W(w, p') - \mathbb{E}_p U(p')\right] + \beta \mathbb{E}_p U(p'). \tag{B.1}$$

The value of being employed is as before:

$$W(w, p) = w + \beta(1-\lambda)\mathbb{E}_p W(w, p') + \beta\lambda \mathbb{E}_p U(p'). \tag{B.2}$$

Given U and Q, let

$$H(p) = \mathbb{E}_p[\mathbb{E}_{p'} Q(p'')] - \mathbb{E}_p U(p'). \tag{B.3}$$

Then, equation (B.1) can be written as

$$U(p) = \tilde{z} + \beta f(q) \left(\frac{w}{1-\beta(1-\lambda)} + H(p)\right) + \beta \mathbb{E}_p U(p'). \tag{B.4}$$

Therefore, for any posted wage $w \in \mathcal{W}(p)$,

$$\frac{w}{1-\beta(1-\lambda)} + H(p) = \frac{U(p) - \tilde{z} - \beta \mathbb{E}_p U(p')}{\beta f(q)}. \tag{B.5}$$

B.2 Firms

As in Rogerson *et al.* (2005), substituting equation (B.5) into equation (10) for w and taking the first order condition with respect to q yields

$$\frac{y(p)}{1-\beta(1-\lambda)} + H(p) = \frac{U(p) - \tilde{z} - \beta \mathbb{E}_p U(p')}{\beta \alpha'(q)}. \tag{B.6}$$

Combine equations (B.5) and (B.6) to obtain

$$\frac{y(p) - w}{1-\beta(1-\lambda)} = \frac{\eta}{\mu\beta(1-\eta)} \left(U(p) - \tilde{z} - \beta \mathbb{E}_p U(p')\right) q^{\eta}. \tag{B.7}$$

Combining this result with the free entry condition,

$$\frac{1-\eta}{\eta} k = \left(U(p) - \tilde{z} - \beta \mathbb{E}_p U(p')\right) q. \tag{B.8}$$

B.3 Elasticity of the vacancy-unemployment ratio with respect to productivity

In the absence of the aggregate shock, the value of Q is simplified to

$$Q = \frac{\beta\lambda}{1-\beta(1-\lambda)} U. \tag{B.9}$$

Therefore, equation (B.3) becomes

$$H = -\frac{1-\beta}{1-\beta(1-\lambda)} U. \tag{B.10}$$

Then, using these equations, the equilibrium conditions given by equations (B.6) and (B.8) can be rewritten as

$$\frac{p-(1-\beta)U}{1-\beta(1-\lambda)} = \frac{(1-\beta)U - [z - c(1)]}{\beta\alpha'(q)} \tag{B.11}$$

and

$$\frac{1-\eta}{\eta}\frac{k}{q} = (1-\beta)U - [z - c(1)], \tag{B.12}$$

respectively. Note that equation (B.11) uses the fact that $y(p) = p$ under a permanent shock. Combining these two equations and that $q = 1/\theta$, one can arrive at

$$p - [z - c(1)] = \frac{1-\eta}{\eta} k \left[\theta + \frac{1-\beta(1-\lambda)}{\beta\mu(1-\eta)}\theta^{1-\eta}\right]. \tag{B.13}$$

As before, by taking logs and differentiating the result with respect to $\ln p$ while taking into account the steady-state normalization $\theta = 1$ and the fact that $\tilde{\eta} = \eta$,

$$\epsilon^{\mathrm{F}}_{\theta,p} = \frac{d\ln\theta}{d\ln p} = \frac{p}{p - [z - c(1)]} \times \frac{\frac{1}{1-\tilde{\eta}}\frac{1-\beta(1-\lambda)}{\beta\mu} + 1}{\frac{1-\beta(1-\lambda)}{\beta\mu} + 1}. \tag{B.14}$$

Given the normalization, $\mu = f(q)$. Thus,

$$\epsilon^{\mathrm{F}}_{\theta,p} = \frac{p}{p - [z - c(1)]} \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})} + 1}{\frac{1-\beta(1-\lambda)}{\beta f(q)} + 1}. \tag{B.15}$$

B.4 Elasticity of the profit with respect to productivity

Combining the free entry condition $k = \beta\alpha(q)\frac{p-w}{1-\beta(1-\lambda)}$ with equation (B.15), the elasticity of a firm's profit with respect to productivity is given by

$$\frac{d\ln(p - w^{\mathrm{F}})}{d\ln p} = \frac{p}{p - [z - c(1)]} \times \underbrace{\frac{\frac{1-\beta(1-\lambda)}{\beta f(q)} + 1 - \tilde{\eta}}{\frac{1-\beta(1-\lambda)}{\beta f(q)} + 1}}_{L}. \tag{B.16}$$

This is smaller than the one found in equation (A.15) (also see the discussions at the end of Appendix A.6).

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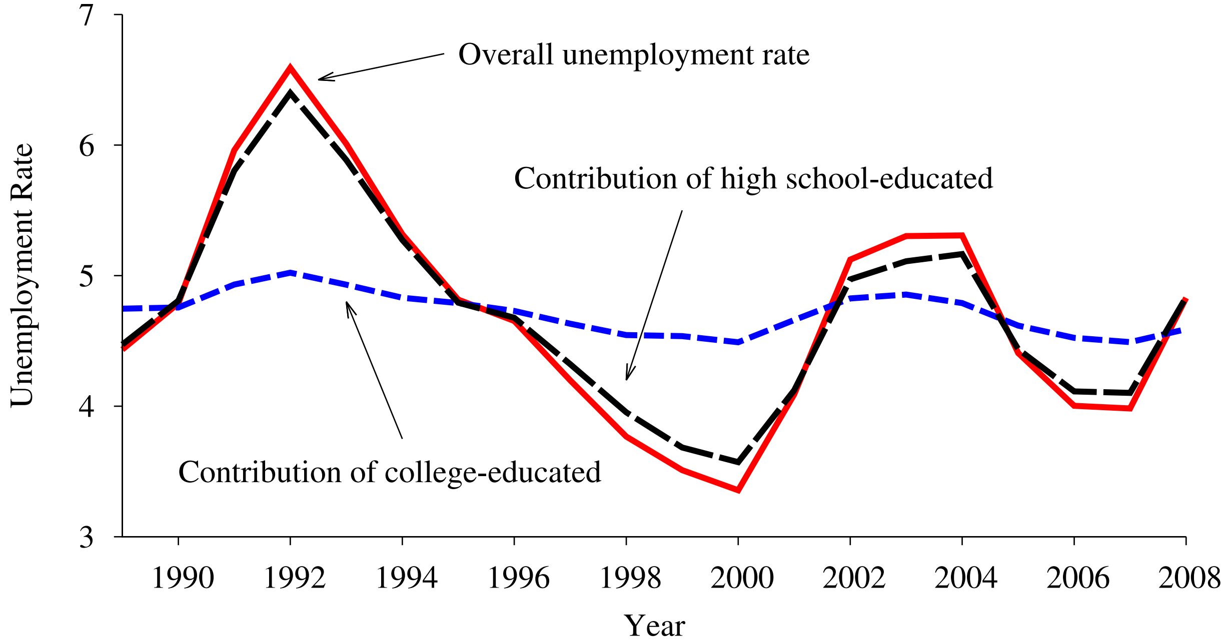
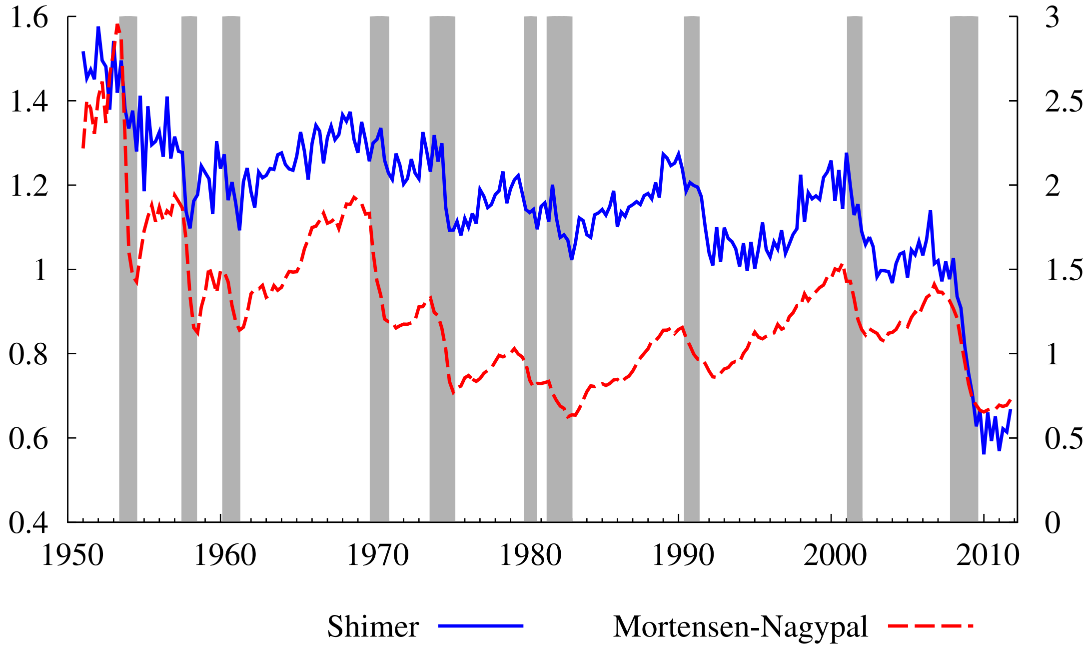
Figure 1: Decomposition of Variation of Aggregate Unemployment



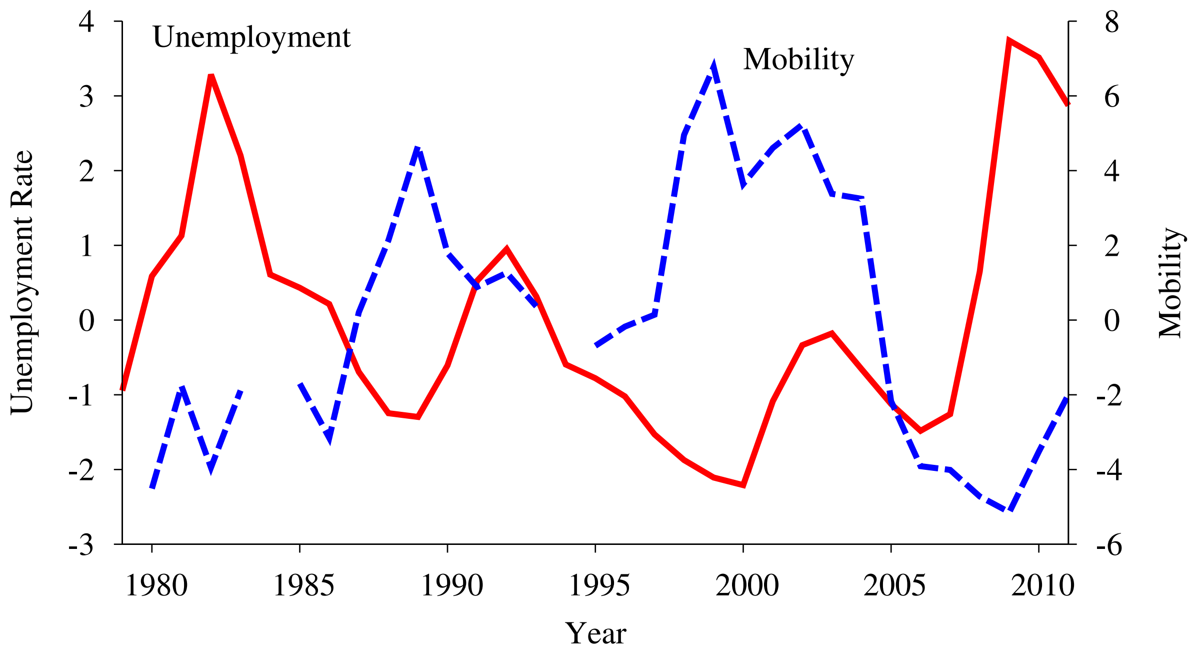
Notes: 'Contribution of college-educated' measures that portion of the cyclical variation in the overall unemployment rate that can be attributed to college educated individuals. Specifically, it computes a hypothetical aggregate unemployment rate that holds the unemployment rate of high school-educated individuals fixed at its sample mean. Similarly, 'Contribution of high school-educated' computes a hypothetical unemployment rate holding the unemployment rate of college-educated at its sample mean. This figure shows that aggregate unemployment fluctuations are mainly driven by unemployment of less educated workers. The coefficient of variation of these two time series over the sample period are 0.035 (contribution of college-educated) and 0.154 (contribution of high school-educated) whereas the coefficient of variation of overall unemployment is 0.182. In other words, unemployment of the less educated group accounts approximately 85% of aggregate unemployment variation over the sample period. The series are constructed from the Current Population Survey of the Bureau of Labor Statistics, which is available from the NBER website. The sample includes adult civilians aged 20-65 years who are in the labor force.

Figure 2: Average Search Intensity



Notes: "Shimer" corresponds to average search intensity as measured by equation (1) (left-hand axis) while "Mortensen-Nagypal" refers to search intensity as given by equation (2) (right-hand axis). Shaded areas are NBER-determined recessions.

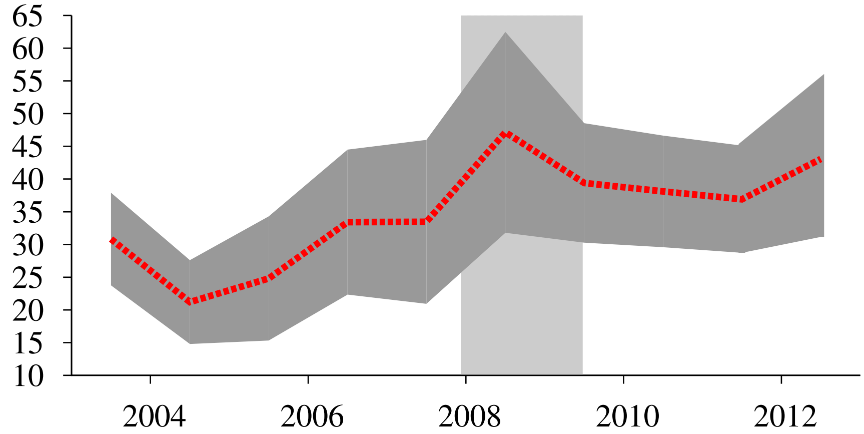
Figure 3: Mobility and the Unemployment Rate



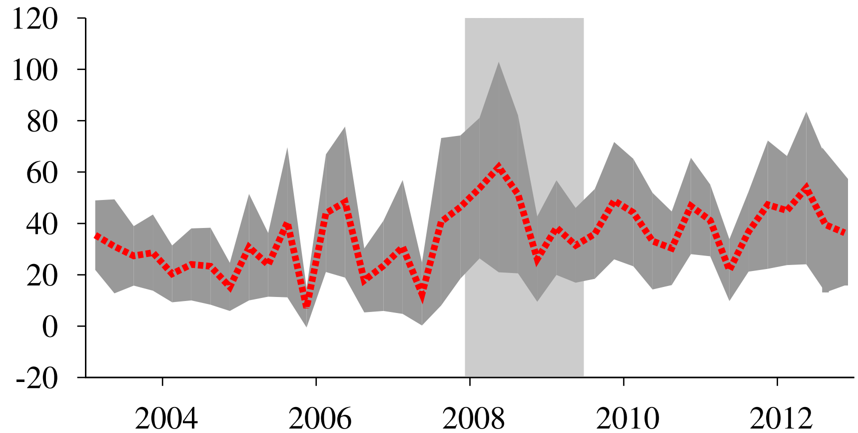
Notes: The unemployment rate is measured as an average from March of the year to February of the following year. The mobility data is computed similarly using March CPS. The CPS did not record mobility in 1985 and 1995. The correlation coefficient of the two series is -0.610 at the significance level 0.003.

Figure 4: Time Spent on Job Search by the Unemployed

(a) Annual



(b) Quarterly



(c) Monthly

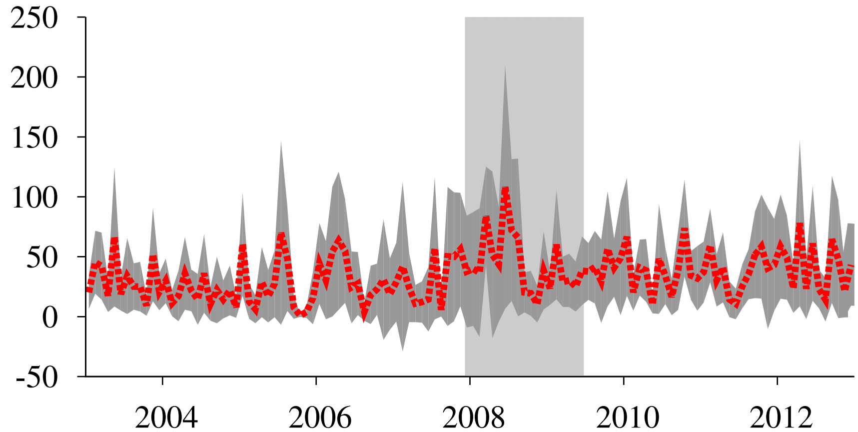


Figure 5: The Impact of a Permanent Productivity Change

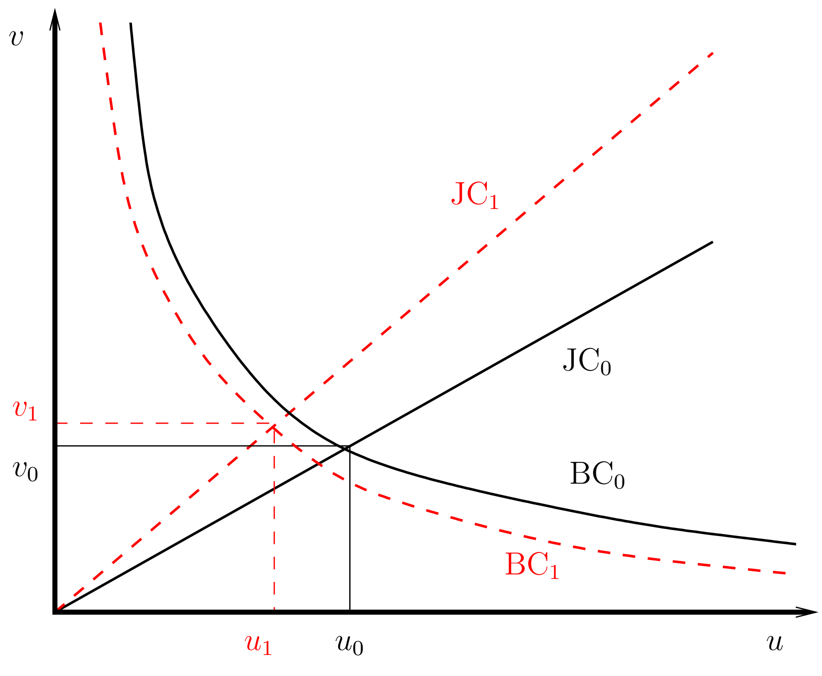


Table 1: Reason for Moving, %

Variables	all	inter-county	interstate
Family	25.24	24.15	22.93
Employment	19.47	36.56	44.04
Housing	34.94	19.47	14.58
Other	20.35	19.82	18.46

Notes: "All" refers to all moves associated with moving into a different house/residential unit. Reasons labeled "other" include attending or leaving college, change of climate and health. The main sample includes adult civilians ages 16-64 who are in the labor force.

Table 2: Unemployment by Mobility Status (%)

Overall unemployment	5.614%
Unemployment among stayers	5.519%
Unemployment among movers	9.150%
Difference, δ	3.497%
	(0.0020)

Notes: The employment and mobility statuses are measured from the CPS of 1980-2012. The unemployment rates and their differences are reported in percentages. δ measures unemployment of movers relative to that of stayers while controlling for individuals' observed characteristics as well as year and state effects by using equation (3). The standard error is in parenthesis.

Table 3: Calibration Targets of the Benchmark Model

	Model	Data
unemployment, u	0.057	0.057
the elasticity of matches w.r.t. vacancies, $\epsilon_{M,v}$	0.544	0.544
average job search time relative to work hours, $\mathbb{E}(\chi s^{\gamma})/(z-b)$	0.084	0.084
the elasticity of time spent on job search w.r.t. benefits, $\epsilon_{s,b}$	-1.907	-1.907

Table 4: Parameters of the Benchmark Model

Parameter	Value	Description
β	0.9992	the time discount factor ($= 1/1.04^{1/48}$)
λ	0.0083	the separation rate ($= 0.1/12$)
ϱ	0.9903	persistence of the productivity shock
σ	0.0033	the standard deviation of the innovation to productivity
μ	0.1438	the coefficient of the matching technology
k	0.0343	the vacancy creation cost
b	0.3	unemployment insurance benefit
z	0.9212	flow utility of unemployment when search intensity is zero
χ	0.0526	the average search cost
η	0.2150	the parameter of the matching technology
γ	2.3858	the power of the search cost function

Table 5: Select Business Cycle Moments

		u	v	v/u	s	p
US Data:						
Standard deviation		0.128	0.140	0.262	0.106	0.013
Autocorrelation		0.882	0.905	0.902	0.865	0.768
Cross-correlation	u	1	-0.912	-0.976	-0.988	-0.252
	v		1	0.980	0.844	0.420
	v/u			1	0.934	0.347
	s				1	0.169
	p					1
Benchmark Model:						
Standard deviation		0.103	0.126	0.216	0.091	0.013
Autocorrelation		0.829	0.612	0.760	0.761	0.765
Cross-correlation	u	1	-0.773	-0.929	-0.929	-0.919
	v		1	0.953	0.953	0.922
	v/u			1	1	0.977
	s				1	0.977
	p					1
Lower z:						
Standard deviation		0.035	0.034	0.067	0.028	0.013
Autocorrelation		0.788	0.674	0.764	0.764	0.765
Fixed Effort, benchmark η:						
Standard deviation		0.019	0.082	0.100		0.013
Autocorrelation		0.822	0.732	0.763		0.765
Fixed Effort, $\eta = 0.544$:						
Standard deviation		0.055	0.052	0.105		0.013
Autocorrelation		0.783	0.685	0.763		0.765

US Data: All moments are based on quarterly data, 1951Q1–2011Q4, logged and HP-filtered with a smoothing parameter of 1600. Unemployment, u, corresponds to the civilian unemployment rate; vacancies are given by a combination of the Conference Board's Help-Wanted Index and work by Barnichon (2010); search effort, s, is computed from equation (2); and productivity, p, is measured by output per person for the non-farm business sector (BLS variable PRS85006163). *Models:* We report averages over $20,000$ replications of the model economy with 244 quarters, after discarding the first $1,000$ weeks of data.