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Worker Search Effort as an Amplification Mechanism☆

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Abstract

It is well known that the Diamond-Mortensen-Pissarides model exhibits a strong trade-off between cyclical unemployment fluctuations and the size of rents to employment. Introducing endogenous job search effort reduces the strength of the trade-off while bringing the model closer to the data. Ignoring worker search effort leads to a large upward bias in the elasticity of matches with respect to vacancies. Merging the American Time Use Survey and the Current Population Survey, new evidence in support of procyclical search effort is presented. Average search effort of the unemployed is subject to cyclical composition biases.

Keywords: Variable Search Effort, Unemployment and Vacancies, Beveridge Curve, Search Intensity, Time Use

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1. Introduction

The Diamond-Mortensen-Pissarides (DMP) model of search and matching is a widely accepted model of equilibrium unemployment. Shimer (2005) argues that the textbook version of the model underpredicts, by an order of magnitude, the cyclical variability in key labor market variables that are central to this theory, namely vacancies and unemployment; similar results are also found in Andolfatto (1996) and Merz (1995). In this paper, worker search effort is introduced as in Pissarides (2000, Ch. 5). As a result, workers can take direct action to affect the outcome of their labor market search, a channel absent from most previous quantitative studies of the DMP model, an exception being Merz (1995). Search effort by the unemployed can serve as a strong amplification mechanism.

An innocuous change is made to the DMP framework, dropping what Rogerson et al. (2005) refer to as the black box of the Nash bargaining solution determination of wages in favor of competitive search which entails wage posting by firms and directed search on the part of the unemployed; see Moen (1997) and Rogerson et al. (2005).[1] Wage posting is motivated by the following considerations. First, as documented by Hall and Krueger (2012), wages of newly-hired workers with less than college education are predominantly determined through wage posting, not bargaining. Second, working with data from the Current Population Survey (CPS) reveals that over 85% of the cyclical variation in unemployment is due to individuals with less than college education; see Figure 1. Third, on the theoretical side, competitive search with wage posting avoids having to take a stand on how variable search effort enters bargaining.

Figure 1 here.

Workers' search cost is central to this paper. This cost function is governed by two

[1] Adopting competitive search is innocuous in the sense that the bulk of the literature that employs Nash bargaining imposes parameter restrictions that deliver constrained-efficient allocations; competitive search of the variety used here delivers the same constrained-efficient allocations as Nash bargaining.

parameters: a scale or level parameter, and a curvature parameter. The benchmark calibration chooses the scale parameter such that the flow value of being unemployed, net of search costs, is 71% of productivity based on the detailed analysis of Hall and Milgrom (2008), and imposes a quadratic search cost, a restriction that is consistent with the available empirical evidence (see Yashiv (2000) who used Israeli data, Christensen et al. (2005) who used micro data from Denmark, and Lise (2013) who used data on white males in the U.S.) and recent calibration work (Nakajima, 2012). Under this calibration, the model accounts for nearly 40% of the variability of vacancies, unemployment, and the vacancy-unemployment ratio. Endogenous search effort is an important ingredient of the model, and its effects work most strongly through unemployment, and so the vacancies-unemployment ratio. Too see this, the model is also solved with fixed search intensity. In this case, volatility of labor market variables drops sharply, and the model exhibits a very steep, thin, short streak of points defining its Beveridge curve, measured at an annual frequency. In contrast, when search effort is endogenous, the Beveridge curve is much flatter, more spread out, and stretched in the sense that it covers a wider range of values for vacancies and unemployment.

In the literature, match surplus, defined as productivity less the flow value of unemployment, is a key determinant of the success of the DMP model (Mortensen and Nagypál, 2007; Hagedorn and Manovskii, 2008). An interesting analytical finding presented below is that in the presence of endogenous worker search effort, labor market volatility is mainly determined by gross flow income while unemployed (relative to productivity), which is consistent with Hagedorn and Manovskii (2008). Further, in the model with search effort, match surplus is higher because of a lower net flow income while unemployed. Thus, endogenous worker search effort reduces the strength of the severe trade-off between the match surplus and cyclical fluctuations in unemployment and vacancies. In the benchmark calibration, match surplus is 29% of productivity. Relative to a model with fixed search effort, this calibration more than doubles the volatility of labor market variables.

To understand the role of search effort in the model, first consider the model without an effort dimension. As described in Shimer (2005), an increase in productivity increases the value of a match. As a consequence, firms post more vacancies which boosts workers' job finding rate, raising their outside option (the value of being unemployed). The net result is that wages rise, eating up much of the gain received by firms associated with the increase in productivity, thereby lowering the response of vacancies. *With effort*, the productivity increase leads the unemployed to search more intensively which dampens the rise in the value of being unemployed, and so the increase in the wage. In this case, the smaller increase in the wage leaves more of the surplus for firms, thus amplifying the response of vacancies. There is a sort of virtuous circle in which the increase in vacancies leads workers to search more which leads to more vacancies, and so on.

The results in this paper would be vacuous if the choice of the search cost function were unconstrained. Section 5 shows analytically that the properties of this cost function are constrained by the elasticity of the matching function with respect to the vacancy-unemployment ratio. Empirical plausibility then places strong restrictions on the search cost. While these analytical results point to the importance of variable search intensity in the model, highly elastic search intensity would likely be inconsistent with the data on unemployment and vacancies, and particularly the elasticity of matches with respect to the vacancies-unemployment ratio.

A key prediction from standard search models with endogenous search effort is that effort is procyclical. Introspection provides little help in determining the plausibility of this result. Search effort will be countercyclical if, during recessions, the unemployed are motivated to search more intensively in the face of an otherwise falling job-finding rate. Alternatively, recessions are lousy times to be looking for a job; since the returns are low, search effort "should be" procyclical. Section 2 explores the evidence concerning the cyclical properties of search effort. Direct evidence is sparse and mixed. Shimer (2004) used the number of search methods from the CPS; he found that this measure of search effort is countercyclical. More recently, Mukoyama et al. (2014) also conclude that search effort is countercyclical using a combination of job search time in American Time Use Survey (ATUS) and the number of search methods in CPS. Countering these works, Tumen (2014) shows, empirically, why the number of search methods is a poor proxy for search effort. He proposes using the number of search methods per week unemployed as an alternative; this measure is procyclical. Using time use data, DeLoach and Kurt (2013) argue that search effort is procyclical.

We introduce new evidence by merging the ATUS and CPS data. Since the ATUS sample is a subset of individuals completing a set of interviews for the CPS, the unemployed can be divided into two groups, short- and long-term unemployed, depending on whether they were employed at their final CPS interview. While search time of the long-term unemployed is slightly and insignificantly countercyclical, that of the short-term unemployed is strongly and significantly procyclical. This result suggests that there may be an important composition bias in average time spent on search. A key finding is those workers who had high wages and hours subsequently spend more time searching for a job during an unemployment spell. Combining this result with the finding that high wage and high hours workers have more cyclical separations and job-finding rates (Bils et al., 2012) suggests that the distribution of search time by workers changes systematically over the business cycle – which may account for the finding that *average* search time of the long-term unemployed is insignificantly countercyclical. In other words, since high wage, high hours workers spend more time on search during an unemployment spell and the share of such workers among the long term unemployed will rise during recessions, *average* search time of the long term unemployed can move countercyclically owing to the change in the composition of the unemployment pool.[2]

Therefore, in order to establish the cyclicality of job search time of a typical unemployed person, one must control for the past wage and hours. In this regard, focus on the short term unemployed, for whom data on both wages and hours is available; for this group, job search time remains strongly procyclical after controlling for the above composition effect.

Section 2 also surveys less direct evidence of the cyclical properties of search effort by the unemployed. Krueger and Mueller (2010) find that individuals with higher expected wages search more; Section 2 shows why this is consistent with procyclical search effort. The micro-labor literature (early works include Katz and Meyer, 1990; Meyer, 1990) finds empirical evidence that the exit rate from unemployment falls with the level of unemployment benefits. In this literature, this result is interpreted to mean that the unemployed alter their search behavior. In the DMP model, changes in unemployment insurance and changes in wages have the same effect, although with opposite signs. Thus, the micro-labor literature is also consistent with procyclical search effort.

Yashiv (2000) appears to be the only paper that estimates the matching technology when search intensity of the unemployed is endogenous; he used Israeli data.[3] In general, ignoring search intensity may be an important oversight. The results in Section 7 show that neglecting search intensity introduces a large upward bias in the elasticity of the number of matches with respect to vacancies; this result is consistent with the empirical work of Yashiv. For the benchmark calibration, ceteris paribus, omitting search effort would lead one to erroneously

[2]Suppose that there are only two types of searchers: low (wage, hours, search) and high (wage, hours, search). During expansions, the relative shares are 80-20; during recessions, 50-50. Suppose low types spend 30 minutes per day searching; high types 60. Then, average search time during an expansion is $.8 \times 30 + .2 \times 60 = 36$; during a recession, $.5 \times 30 + .5 \times 60 = 45$. This example shows that average search time can be countercyclical even when search time of each group is independent of the cycle.

[3]Yashiv's (2000) principal contributions are to estimate the various frictions in the matching process, including the matching function, firm search, and worker search. He does not perform a quantitative evaluation of the model like that contained herein, nor does he provide analytical results as we do. Christensen et al. (2005) and Lise (2013) also estimate search cost functions, but co-mingle search by the unemployed with on-the-job search; neither do they jointly estimate the search cost and matching functions.

conclude that a 10% increase in vacancies would increase the number of matches by more than 5% whereas the actual impact is less than 1%. Such a discrepancy should make one cautious in interpreting results from equilibrium search and matching models with fixed search intensity, particularly when quantitatively evaluating the effects of alternative public policies such as the effects of unemployment benefits and employment subsidies.

Another, even more important implication of the findings in Section 7 concerns the Nash bargaining parameter, which is central to standard search and matching theory. In the literature, the Nash bargaining parameter is usually inferred from data on unemployment and vacancies (Shimer, 2005; Mortensen and Nagypál, 2007). Specifically, guided by the Hosios (1990) condition, a worker's bargaining power is set to the elasticity of matching function with respect to unemployment. The results in Section 7 suggest that the common method of estimating bargaining power exhibits a strong downward bias. For example, the numerical results show that when the elasticity of matching with respect to unemployment is 0.46, the worker's bargaining power parameter required to achieve the constrained efficient allocation is not 0.46, but rather 0.91. Conversely, picking the bargaining parameter based on the measured elasticity of the matching function with respect unemployment or vacancies cannot always guarantee constrained efficiency. These results point to one of the benefits of adopting competitive search instead of Nash bargaining determination of wages: For the standard DMP model, the allocations associated with competitive search are always efficient; see Moen (1997). Moreover, the above bias in the matching technology combined with the Hosios (1990) condition has an important quantitative implication on volatility of the labor market. For example, Hagedorn and Manovskii (2008) show that a smaller bargaining power for a worker means a weaker response of the wages to productivity. Therefore, the downward bias in the bargaining power of a worker implies a less volatile wage (also see Appendix C.4).

The outline of the rest of the paper is as follows. Section 2 surveys the literature on the cyclical properties of search effort as well as presenting some evidence on its cyclicality.

Section 3 presents a dynamic, stochastic model of equilibrium unemployment incorporating endogenous search intensity into a competitive search model. Section 4 presents key analytical results characterizing the equilibrium. Section 5 explores the steady-state properties of the model. The model is calibrated and simulated in Section 6, establishing the model's business cycle properties. Implications of endogenous search intensity on the aggregate matching technology are discussed in Section 7. Section 8 concludes.

2. Empirical evidence on the cyclical properties of search effort

This section starts by briefly discussing the existing literature; what little direct evidence there is on the cyclicality of search effort of the unemployed is mixed. Then, we present new evidence on the cyclicality of search effort by merging the ATUS and CPS data. This evidence shows that search effort by the short-term unemployed is strongly procyclical. New evidence also suggests that average search effort is subject to compositional biases over the business cycle. Finally, some less direct evidence is reviewed regarding the cyclicality of search intensity that comes from the empirical micro-labor literature. This indirect evidence also supports the notion that search effort is procyclical.

2.1. Brief literature review

Shimer (2004) is an early and influential work trying to infer the cyclical properties of search effort of the non-employed. From the CPS, Shimer uses the number of search methods as a proxy for search effort; by this measure, search effort is countercyclical. Tumen (2014) questions Shimer's measure of search effort. After controlling for individual characteristics, Tumen finds that an increase in the number of search methods *reduces* the probability of exiting unemployment, a result that is inconsistent with search being a costly activity. Tumen suggests using the number of search methods per week unemployed as an alternative measure of search effort; he finds that this measure is strongly procyclical. As Elsby et al.

(forthcoming) point out, countercyclical search effort of workers is difficult to reconcile with movements in the Beveridge curve during and after the Great Recession.

The American Time Use Survey is a relatively new source of information on time spent on job search. To the extent that time on job search corresponds to search effort, the data seem ideal. Two of the more important limitations of the ATUS are its relatively short length (it is only available since 2003 which means it covers only one business cycle), and its cross-sectional nature (participants for the ATUS are drawn from individuals who have recently completed their final interview for the CPS, and so one gets no information on how an *individual's* search time varies over time).

Figure 2 presents average search time of the unemployed (hereafter simply referred to as "average search time") based on the ATUS data. Average search time rose from 33.5 minutes per day just before the Great Recession to 47.1 minutes per day, suggesting that average search time is countercyclical. However, there is considerable uncertainty around these means, a feature of the data that has received relatively little attention in the literature. In particular, the 13.7 minute per day rise in search time (from 2007 to 2008) is within the two standard deviation bound for 2008; see the *ATUS User's Guide* for the methodology for computing error bounds. So, focusing solely on the aggregate series, it simply is not clear that search time actually went up at the beginning of the Great Recession.

Figure 2 here.

An important consideration in interpreting the ATUS data is that the characteristics of the unemployment pool likely changes over the cycle. Thus, to infer the behavior of a typical unemployed person, it is necessary to control for individual characteristics. DeLoach and Kurt (2013) perform such an analysis and find that job search time among the unemployed is procyclical. They also find that a reduction in individuals' wealth leads them to increase their search. Mukoyama et al. (2014) use data on the number of search methods from the

CPS to infer what average time use was prior to the ATUS. Like DeLoach and Kurt (2013), Mukoyama et al. are careful to control for individual fixed effects in their empirical work. They too find that losses in wealth increase search time, but conclude that search effort is countercyclical, in stark contrast to DeLoach and Kurt. While Mukoyama et al.'s attempt to lengthen the time span of the time use data is laudable, their use of the number of search methods to do so subjects them to the same critique that Tumen (2014) levels at Shimer (2004).

In addition to Tumen's (2014) criticism, there are two other important issues concerning the link between time spent on job search and the number of search methods. The first is that it is hard to establish a sufficiently strong link at the individual level between search time and the number of methods used in search. Appendix A shows that OLS regressions of search time on the number of search methods delivers a very low R^2, well below 10%, even after controlling for the individual level characteristics. This is, perhaps, due to the fact that the ATUS uses diary entries for a particular day, say June 1, to measures time on job search, whereas the number of search methods in the ATUS covers activities over the previous four weeks – in this example, most of May. The second issue is whether such an *individual-level* link can be used to infer the cyclical behavior of average search time using the average number of search methods. Indeed, Appendix A shows that despite the positive individual-level link between the two variables, they do not move in the same direction over the business cycle.

2.2. New evidence from ATUS and CPS

While the ATUS data is cross-sectional (a household is in the ATUS but once), it can be combined with the CPS to give it some panel data-like features, which allows us to further control for the characteristics of the unemployed in ATUS.

Short- versus long-term unemployed

Of particular interest at this point is to gauge the importance of differences in the behavior of the short- and long-term unemployed. Someone will be classified as short-term unemployed if, at their last CPS interview they reported being employed. Given the timing of the CPS and ATUS interviews, such a person will have been unemployed for no more than five months. Those who report being unemployed at both their ATUS and final CPS interview are said to be long-term unemployed. The following regression is run:

$$s_{i,t} = \phi_t + \tilde{\beta}\mathbf{X}_i + \epsilon_{i,t}, \tag{1}$$

where $s_{i,t}$ is job search time of person i in year t, ϕ_t is a dummy for year t, $\mathbf{X}_i$ contains age, education, dummies for race and sex, and $\epsilon_{i,t}$ is the error term. Here, ϕ_t gives average search time, by year, after controlling for a variety of individual characteristics.

Next, compute the correlation between the estimate of search time ϕ_t and the Hodrick-Prescott filtered vacancy-unemployment ratio θ_t (see Table 4), the traditional measure of labor market conditions. While correlations are computed for all unemployed, short-term unemployed and long-term unemployed, only the correlation for the short-term unemployed is significant (its p value is 0.02); see Table 1. The correlation, 0.72, is large and positive. The interpretation of the sign is that the short-term unemployed raise their job search time when labor market conditions improve, indicating that their search effort is procyclical. This finding conforms with the prediction in the model that search effort is positively related to the vacancy-unemployment ratio; see (12), below.

Table 1 here.

Among the long-term unemployed, the correlation between search time and the vacancy-unemployment ratio is -0.23, although it is insignificant with the p-value 0.53. Based on the results of the long-term unemployed, one could argue that search time is acyclical or countercyclical. However, the difference between the short- and long-term unemployed suggests an important composition effect among the unemployed.

Composition bias in average job search time

There are good reasons to believe that there are other changes in the composition of the unemployment pool that drive average time spent on search. To motivate this line of thought, suppose that there are two types of workers, high and low, and high types spend more time looking for a job than low types. For the sake of argument, suppose that their search times are constant. If, during a recession, the fraction of high types in the unemployment pool rises, then average search time will increase even though individuals' search time is unchanged. Observing the average, one would erroneously conclude that search time (effort) is countercyclical; see footnote 2 for a numerical example. The question now is whether or not this is a plausible mechanism. It is. To start, Bils et al. (2012) show that labor market transitions are much more cyclical among high-wage and high-hours workers, implying that the share of these workers in the unemployed pool is countercyclical. The link between search time, on the one hand, and wages and hours, on the other, can be established directly by merging search time in the ATUS with wages and hours in the CPS. Consider the following regression:

$$s_{i,t} = \phi_t + \tilde{\beta}\mathbf{X}_i + a_w w_i^{CPS} + a_h h_i^{CPS} + \epsilon_{i,t}, \tag{2}$$

where w_i^{CPS} and h_i^{CPS} are, respectively, person i's log weekly real wage and log weekly hours from that person's CPS interview. Wages are adjusted by the Consumer Price Index for All Urban Consumers. The estimate of a_w is 1785.514 with the standard error 847.061 (p value of 0.04) and of a_h is 13.052 with the standard error 6.908 (p value of 0.06). When the log wage is dropped from the regression, $a_h = 13.478$ with the standard error 6.881 (p value of 0.05). So, high-wage and high-hours workers spend more time on job search during a subsequent unemployment spell. This result suggests that changes in the composition of the unemployment pool may impart a countercyclical bias to observed average search time.

Since the effects of past wages and hours on search time should also apply to the long-term unemployed, the effects of this bias will also apply to this group.

Cyclicality of search time of a typical unemployed worker

Above, we argued that the relative share of workers with high wages and high hours may impart a countercyclical bias to observed average search time. Thus far, the discussion of the fluctuations in search effort have focused on the changes in the composition of the pool of searchers. The analysis above does not address the cyclicality of search time of a *typical* unemployed worker. For example, search time of workers with higher wages and higher hours could remain constant over the cycle while their share in unemployment moves countercyclically due, perhaps, to a greater procyclicality in job openings for such workers. So, when one looks at *average* search time over the cycle, one may be picking up cyclical changes in the composition of the unemployment pool with individual search time constant. Then, the natural question is whether (besides these compositional shifts) 'identical' individuals alter their search effort over the business cycle. Indeed, since, in the model considered below, all unemployed workers are ex ante identical, the economically relevant shifts are in the search time of observationally identical workers over the business cycle.

As mentioned above, the ATUS provides information on an individual's search time *at a point in time* and so cannot be used directly to get at how an *individual's* search time varies over time. However, the ATUS data can be linked back to the CPS which provides information on the previous wages and hours of short-term unemployed individuals, and this information can be used to gain some insight into the cyclicality of search time of a typical worker. Specifically, look at search time after controlling for the wage and hours (in addition to the demographic variables and education). Such search time is given by the time dummies in (2). The correlation between these new time dummies and the vacancy-unemployment ratio is 0.72 with the significance level of 0.02, suggesting that search time remains procyclical at the individual level even after controlling for the wage and hours.

The elasticity of job search time to the vacancy-unemployment ratio

Next the elasticity of search time with respect to the vacancy-unemployment ratio, θ, is measured. For this purpose, run the following regression:

$$s_{i,t} = a_0 + \tilde{\beta}\mathbf{X}_i + a_\theta \theta_t + \epsilon_{i,t}, \tag{3}$$

where a_θ measures the impact of a percentage increase of the vacancy-unemployment ratio on search time. Then, as in Krueger and Mueller (2010), the elasticity is computed by dividing the coefficient estimate by the mean of the dependent variable. As before, consider the following three samples of the unemployed: *all*, *short-term*, and *long-term*.

For the short-term employed, also consider the following regression to control for the wage and hours:

$$s_{i,t} = a_0 + \tilde{\beta}\mathbf{X}_i + a_w w_i^{CPS} + a_h h_i^{CPS} + a_\theta \theta_t + \epsilon_{i,t}. \tag{4}$$

Table 2 summarizes the results for the different samples. The short-term unemployed sample is the only one for which the coefficient on the the vacancy-unemployment ratio, a_θ, is significant at the 5% level. The implied elasticity of search time with respect to the vacancy-unemployment ratio is either 0.516 or 0.540 depending on whether one controls for the wage and hours. It is reassuring that data generated from the model, aggregated to an annual frequency, gives an elasticity of 0.501.

Table 2 here.

To give an idea of the magnitude of this elasticity, consider the effect of changing the vacancy-unemployment ratio by 26.4%, which is the standard deviation of the vacancy-unemployment ratio (see the upper panel of Table 4). Then, time devoted to job search among the short-term unemployed will increase by between 13.6% and 14.3%. These numbers are slightly greater than the volatility of search intensity implied by the model below. These comparisons between the model and U.S. data suggest that the model is doing a reasonably good job of capturing this dimension of the data.

2.3. Further discussion

Findings like the above point to the importance of micro studies in understanding the cyclicality of search intensity. For example, Krueger and Mueller (2010) provide indirect evidence that search effort is procyclical. Using data on time spent on job search from the ATUS, they find that search time increases with a worker's expected wage. While the aggregate wage is only mildly procyclical, Solon et al. (1994) show that individual wages are strongly procyclical, the difference being due to a composition bias. Since recessions are times during which workers have lower expected wages, the Krueger and Mueller evidence suggests that time spent on job search by the unemployed is likely procyclical.

There is a sizable micro-labor literature on the responses of the unemployed to the policy parameters of unemployment insurance (UI) programs; important early contributions include Katz and Meyer (1990) and Meyer (1990). Some common findings in this literature are: holding fixed the number of weeks of unemployment, the probability of exiting unemployment falls with the replacement rate (the UI benefit divided by the previous wage), and rises sharply around the time that an unemployed individual exhausts his/her benefits.[4] These empirical regularities are taken as *prima facie* evidence that the unemployed adjust their search effort in response to these UI program policy parameters. This interpretation of the evidence is typically justified with reference to a search model with endogenous search intensity. Using this evidence to make inferences about the cyclicality of search effort involves a couple of steps. To start, in this model, an increase in UI benefits has the *same effect* as

[4] Another dimension of UI generosity is the duration of benefits, an aspect of policy that has received attention following the Great Recession in light of the extent of the increase in the maximum benefit period (from 26 to 99 weeks) as well as the severity of the recession on labor markets. Both Rothstein (2011) and Farber and Valletta (2013) find that extended benefits had a small, but statistically significant, effect on the exit rate from unemployment, and raised the average duration of unemployment. However, Hagedorn et al. (2013) point out that such work ignores the general equilibrium effects on vacancies, and so may understate the impact of such policies. Another general equilibrium channel is that UI-ineligible individuals face less competition as the UI-eligible reduce their search activity; see Marinescu (2014). Using an online job board, she likewise finds a small negative effect of benefit extension on job applications. Curiously, Marinescu also finds little effect of benefits extensions on vacancies.

a fall in the wage. The next link in the chain of reasoning is to again note that individual wages are highly procyclical (Solon et al., 1994). Therefore, the micro-labor evidence on the effects of changes in UI benefits provides indirect evidence that search effort is procyclical.

3. Model

The economy is populated by a measure one of infinitely-lived, risk-neutral workers and a continuum of infinitely-lived firms. Individuals are either employed or unemployed.[5] An unemployed worker looks for a job by exerting variable search effort. The cost of searching for a job depends on how intensively the worker searches. Let s_i be the search intensity of worker i. The cost of s_i units of search is $c(s_i)$ where c is a twice continuously differentiable, strictly increasing and strictly convex function. Flow utility of unemployed worker i is $z - c(s_i)$. Normalize the cost of search so that $c(0) = 0$, implying that z is flow utility of an unemployed worker who exerts zero search intensity. Flow utility of an employed worker is the wage, w. Workers and firms discount their future by the same factor β.

A firm employs at most one worker. Per-period output of a firm-worker match is denoted by p and evolves according to a Markov transition function $G(p'|p)$ given by $p' = 1 - \varrho + \varrho p + \sigma\varepsilon$, where ε is an *iid* standard normal shock, $0 < \varrho < 1$ and $\sigma > 0$. There is free entry for firms. A firm finds its employee by posting a vacancy, at the per period cost k, when looking for workers. All matches are dissolved at an exogenous rate λ. The matching technology is discussed in Section 3.2.

3.1. Wage determination

Wages are determined via competitive search instead of Nash bargaining. The setup follows Rogerson et al. (2005). Given current productivity, p, a firm decides whether or not to post a vacancy. If it does, the firm decides what wage to offer in order to maximize

[5] Shimer (2004) suggests that labor market participation reflects search effort. We follow the usual practice in the literature in abstracting from flows in and out of the labor force.

its expected profits. An unemployed worker directs her search towards the most attractive job given current aggregate labor market conditions. Let $\tilde{w}$ denote the expected present discounted value of a wage stream offered by a vacant job which is fully characterized by $(p, \tilde{w})$. Let $\mathcal{W}(p)$ denote the set of present discounted values associated with wage streams posted in the economy when aggregate productivity is p.

3.2. Matching technology

Matching between firms and workers operates as follows. Let $s_{i,j}$ denote search effort by unemployed worker i for job type $j = (p, \tilde{w})$ where it is understood that $s_{i,j}$ can be non-zero for at most one j. (There is no on-the-job search.) Since a worker searches for at most one type of job, $s_i = \max_j\{s_{i,j}\}$. Let u_j denote the number of unemployed workers searching for a type j job. Let S_j denote the total search intensity exerted by these workers. Denote total vacancies of type j by v_j. As in Pissarides (2000, Ch. 5), the total number of matches formed for a particular job type is given by the Cobb-Douglas function, $M_j = \mu {v_j}^{\eta} {S_j}^{1-\eta}$ where $0 < \eta < 1$. The (effective) queue length for a type j vacant job is given by $q_j = S_j/v_j$, and the probability that a particular job is filled is given by $\alpha(q_j) = \mu q_j^{1-\eta}$. The probability that an unemployed worker i finds a job of type j is $f(q_j)s_{i,j}$ where $f(q_j) = \mu/q_j^{\eta}$. Let θ_j denote labor market tightness for a type j job: $\theta_j = v_j/u_j$. For notational brevity, the individual index i is omitted for the rest of the paper.

3.3. Value functions

Let $W(\tilde{w}, p)$ denote the value to a worker of a new job offering $\tilde{w}$ when the current state is p. Let $U(p)$ denote the value of being unemployed. Then, the value of searching for a job offering $\tilde{w}$ when aggregate productivity is p is given by

$$\begin{aligned}\tilde{U}(\tilde{w}, p) \equiv \max_{s_{\tilde{w},p}} \Big\{ & z - c(s_{\tilde{w},p}) + \beta f(q_{\tilde{w},p}) s_{\tilde{w},p} \int W(\tilde{w}, p') dG(p'|p) \\ & + \beta \left[1 - f(q_{\tilde{w},p}) s_{\tilde{w},p}\right] \int U(p') dG(p'|p) \Big\}.\end{aligned} \tag{5}$$

An unemployed worker chooses to search for the job that yields the highest expected utility,

$$U(p) \equiv \max_{\tilde{w} \in \mathcal{W}(p)} \{\tilde{U}(\tilde{w}, p)\}, \tag{6}$$

where it is anticipated that there are a finite number of elements in $\mathcal{W}(p)$.

The value of a new job consists of two main components, the expected present value of the wage stream and the expected value of unemployment upon future separation, $Q(p)$:

$$W(\tilde{w}, p) = \tilde{w} + \int Q(p')dG(p'|p) \tag{7}$$

where $Q(p) = \beta\lambda U(p) + \beta(1-\lambda) \int Q(p')dG(p'|p)$.

Let $Z(p)$ denote the value of the expected output streams of a firm when the current state is p: $Z(p) = p + \beta(1-\lambda) \int Z(p')dG(p'|p)$. Then, the value of a new match to a firm offering $\tilde{w}$ to its employee is given by:

$$J(\tilde{w}, p) = \int Z(p')dG(p'|p) - \tilde{w}. \tag{8}$$

Finally, the value of a vacancy is

$$V(p) = \max_{\tilde{w}} \left\{-k + \beta\alpha(q_{\tilde{w},p})J(\tilde{w}, p)\right\}. \tag{9}$$

The formal definition of the labor market equilibrium is provided in Appendix B.

4. Equilibrium characterization

Since unemployed workers are intrinsically identical and direct their search to the most attractive jobs, the value of unemployment $U(p)$ is common across all workers. Consequently, the non-wage component of the value of employment, $Q(p)$, is also common across jobs.

Workers take the queue length, $q_{\tilde{w},p}$, as given. The first-order condition with respect to search intensity, $s_{\tilde{w},p}$, in (5) is

$$c'(s_{\tilde{w},p}) = \beta f(q_{\tilde{w},p}) \left[W^e(\tilde{w}, p) - U^e(p)\right], \tag{10}$$

where $U^e(p) = \displaystyle\int U(p')dG(p'|p)$ and $W^e(\tilde{w}, p) = \displaystyle\int W(\tilde{w}, p')dG(p'|p)$. As in Rogerson et al. (2005), firms make their wage posting decision taking (10) as given. Specifically, a firm's problem in (9) can be reduced to: $\max_{q_{\tilde{w},p}} \alpha(q_{\tilde{w},p})J(\tilde{w}, p)$ subject to (10). Substituting (10) into the firm's first-order condition, using the fact that $\frac{dJ(\tilde{w},p)}{d\tilde{w}} = -\frac{dW^e(\tilde{w},p)}{d\tilde{w}} = -1$, and the free entry condition, $J(\tilde{w},p) = k/(\beta\alpha(q_{\tilde{w},p}))$, gives

$$\eta q_{\tilde{w},p} c'(s_{\tilde{w},p}) = k(1-\eta). \tag{11}$$

Proposition 1 (**Same jobs**). *Given current productivity, all firms creating a vacancy offer the same level of the present discounted value of wages. (See Appendix B.2 for the proof.)*

Proposition 1, along with the free entry condition, implies that the vacancies created within the same period have the same queue length, that is $q_{\tilde{w},p}$ is unique to productivity p. Then, using (11), one can make the following claim:

Corollary 1 (**Same effort**). *All unemployed workers exert the same search intensity.*

These results are obtained without making any specific assumption on the shape of the wage profile for a given match.[6] Given the uniqueness result, the subscripts of s, q and θ are dropped. Then, (11) can be rewritten as $qc'(s) = k(1-\eta)/\eta$ or, equivalently,

$$\eta s c'(s) = k(1-\eta)\theta. \tag{12}$$

(11) and (12) represent key analytical results. Specifically, they show that in equilibrium, labor market tightness, θ, and search intensity, s, are positively related.

5. Steady state analysis

Here productivity, p, is constant over time. Proceeding as in the previous section, it can be shown (see Appendix B.3) that in equilibrium,

$$p - z = \frac{1-\beta(1-\lambda)}{\beta\alpha'(q)} c'(s) + c'(s)s - c(s). \tag{13}$$

Proposition 2 (**Permanent shock**). *An increase in productivity raises search intensity, the vacancy-unemployment ratio and the job-finding rate. (See Appendix B.4 for the proof.)*

[6] We are grateful to an anonymous referee for directing us toward this equilibrium characterization, which uses transferability of utility between a firm and its employee. In a previous version of the paper, Eq. (11), Proposition 1 and Corollary 1 were obtained by imposing a constant wage within a match.

Given the strict convexity of the search cost function, c, (12) implies that market tightness, θ, is strictly increasing with search intensity, s. More importantly, in light of Proposition 2, (12) suggests that the volatility of the vacancy-unemployment ratio is closely related to the search cost. This relation is quantified in the following section.

5.1. The elasticity of the vacancy-unemployment ratio to productivity

Next the analytical results in Hagedorn and Manovskii (2008) and Mortensen and Nagypál (2007) are extended to the model with endogenous search intensity. Specifically, the elasticity of the vacancy-unemployment ratio to productivity, defined as $\frac{d\ln\theta}{d\ln p}$, is calculated and compared with that in the standard model with fixed search intensity.

Let $\tilde{\eta}$ denote the implied (or empirical) elasticity of the job-finding rate with respect to the vacancy-unemployment ratio; that is, $\tilde{\eta} = \frac{d\ln(f(q)s)}{d\ln\theta}$. Without loss of generality, normalize search intensity to 1. Taking logs in (13) and differentiating the result with respect to $\ln p$, it can be shown that (see Appendix B.6)

$$\frac{d\ln\theta}{d\ln p} = \frac{p}{p-z} \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})} + \left(1 - \frac{c(1)}{c'(1)}\right)\left(1 + \frac{c'(1)}{c''(1)}\right)}{\frac{1-\beta(1-\lambda)}{\beta f(q)} + 1}. \tag{14}$$

Given convexity of the search cost function it follows that $0 < \frac{c(1)}{c'(1)} < 1$ and $\frac{c'(1)}{c''(1)} > 0$, and therefore, $C \equiv \left(1 - \frac{c(1)}{c'(1)}\right)\left(1 + \frac{c'(1)}{c''(1)}\right) > 0$. In steady state, the unemployment rate is $\frac{\lambda}{\lambda+f(q)}$. Given that the average unemployment rate for the U.S. is around 6% (Shimer, 2005), it follows that $\frac{\lambda}{\lambda+f(q)} \simeq 0.06$ which implies $f(q) \gg \lambda$. When the model period is relatively short, the discount factor, β, is close to 1 and so $\frac{1-\beta(1-\lambda)}{\beta f(q)} \simeq \frac{\lambda}{f(q)}$ is much smaller than 1. Further, the observed elasticity $\tilde{\eta} \simeq 0.5$ (Petrongolo and Pissarides, 2001; Mortensen and Nagypál, 2007) and so $\frac{1-\beta(1-\lambda)}{\beta f(q)}\frac{1}{1-\tilde{\eta}} \simeq \frac{\lambda}{f(q)}\frac{1}{1-\tilde{\eta}}$ is also much smaller than 1. The upshot is that the magnitude of the elasticity $\frac{d\ln\theta}{d\ln p}$ is dictated by $\frac{p}{p-z}$ and $\left(1 - \frac{c(1)}{c'(1)}\right)\left(1 + \frac{c'(1)}{c''(1)}\right)$.

Clearly, the magnitude of this elasticity can be made arbitrarily large by assuming a cost function such that $\frac{c(1)}{c'(1)} \ll 1$ and $\frac{c'(1)}{c''(1)} \gg 1$. However, doing so will lead to highly counterfactual implications. Specifically, using the fact that $\frac{d\ln\alpha(q)}{d\ln q} \leq 1$,

$$C < 1 + \frac{c'(1)}{c''(1)} = \frac{1}{1-\tilde{\eta}}\frac{d\ln\alpha(q)}{d\ln q} \leq \frac{1}{1-\tilde{\eta}} \simeq 2.193, \tag{15}$$

where the value $\tilde{\eta} = 0.544$ is obtained from Mortensen and Nagypál (2007). So, the empirical elasticity of the matching function, $\tilde{\eta}$, dictates that C can not be much larger than 2. In fact, if search costs are given by a power function – a commonly-used specification (e.g., Christensen et al., 2005; Nakajima, 2012; and Lise, 2013) – then the value of C is much lower than 2. Specifically, let the function c be given by the following power function:

$$c(s) = \chi s^{\gamma}, \tag{16}$$

where $\chi > 0$ and $\gamma > 1$. Then, $C = 1$, regardless of the values of χ and γ, and (14) becomes

$$\frac{d\ln\theta}{d\ln p} = \frac{p}{p-z} \times \underbrace{\frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})}+1}{\frac{1-\beta(1-\lambda)}{\beta f(q)}+1}}_{K}. \tag{17}$$

For comparison purposes, the above elasticity is also calculated for the model with fixed search intensity ($s = 1$) while the elasticity of the matching function and the unemployment rate are matched with their empirical counterparts. In this case, the elasticity is given by (see Appendix C.3 for derivation)

$$\frac{d\ln\theta^{\mathrm{F}}}{d\ln p} = \frac{p}{p-(z-c(1))} \times K. \tag{18}$$

Given the calibration in Section 6, $\frac{p}{p-z} = 6.463$, $\frac{p}{p-(z-c(1))} = 3.846$ and $K = 1.073$. These numbers imply that $\frac{d\ln\theta}{d\ln p} = 6.938$ while $\frac{d\ln\theta^{\mathrm{F}}}{d\ln p} = 3.702$. So, the elasticity in the two models is determined by either z relative to productivity p (in the case of (17)) or $z - c(1)$ relative to p. Search effort amplifies the elasticity of the vacancy-unemployment ratio with respect to productivity by almost 90%, specifically, $\frac{d\ln\theta}{d\ln p}/\frac{d\ln\theta^{\mathrm{F}}}{d\ln p} = 1.874$.

What is more surprising is that, despite the introduction of search intensity, the elasticity given by (17) coincides with that obtained by Hagedorn and Manovskii (2008) and Mortensen and Nagypál (2007) for the textbook version of the DMP model after imposing the Hosios condition. These results lead to the following two key observations. First, as in the standard model, the elasticity of vacancy-unemployment ratio with respect to productivity in the model with variable search effort is determined by $\frac{p}{p-z}$, which is consistent with Hagedorn and Manovskii (2008). Second, an important difference is that the net flow utility of an unemployed worker in the model with variable search intensity is $z - c(1)$ while that in the standard model (that is, the one without variable search intensity) is simply z. Consequently, the employment surplus can be substantially higher in the model with variable search effort.

In summary, one can generate a sufficient volatility in unemployment and vacancies by using a high gross flow income for the unemployed (that is, high z) while still maintaining a substantial employment surplus through the low net utility for the unemployed, $z - c(s)$.

Given the cost function, C in (15) is 1. A higher value for the elasticity of the vacancy-unemployment ratio with respect to productivity could be obtained by choosing a non-power cost function that brings C closer to its upper bound of around 2.2. We choose not to follow this route, following instead Christensen et al. (2005), Nakajima (2012) and Lise (2013) in using a power function, (16). In fact, the numerical analysis in Section 6 shows that this cost function performs well for moments that are not targeted during the calibration.

5.2. *Main intuition*

Here the main intuition behind the amplifying effect of variable search effort is explained. The specific focus is on how variable search effort amplifies the response of unemployment and vacancies to a shift in productivity. The response of unemployment and vacancies to the cost parameters, such as k, η and χ is discussed later, in Section 7.

There are three main equilibrium channels that are key to understanding the amplifying effect of variable search effort. The first effect arises from the complimentarity of search intensity, reflected in the equilibrium condition in (12). When there is an increase in productivity p, firms create more vacancies and workers search more intensely. The nature of the complimentarity is that as firms increase vacancies, workers search even more, leading firms to post more vacancies, and so on. The second main effect operates through the interaction of search costs and profits. Specifically, an increase in worker search effort lowers the flow utility of unemployment. As a result, the match surplus remains relatively large and firm profits are large enough to encourage a large increase in vacancies (see Appendix B.7 and Appendix C.4). The final effect is a shift in the Beveridge curve arising from the effect of search intensity on the workers' arrival rate of job offers.

How do these effects translate into the equilibrium level of unemployment and vacancies? To answer this question, combine (12) and (13) to obtain

$$p - z = \frac{1 - \beta(1-\lambda)}{\beta\mu} \left(\frac{k}{\eta}\right)^{\eta} \left(\frac{\chi\gamma}{1-\eta}\right)^{1-\eta} s^{(\gamma-1)(1-\eta)} + \chi(\gamma - 1)s^{\gamma}, \tag{19}$$

which shows that search intensity, s, is an increasing function of productivity, p. Combining this result with (12), the vacancy-unemployment ratio, θ, is an increasing function of p. As in Pissarides (2000), the impact of productivity on the vacancy-unemployment ratio is depicted as a counterclockwise rotation of the job creation (JC) curve in the vacancy-unemployment plane in Figure 3. The standard model with fixed effort also exhibits a rotation of the JC curve, but not as large as with endogenous search effort (see Appendix C.4).

Figure 3 here.

On the other hand, changes in search intensity will shift the *theoretical* Beveridge (TB) curve given by $\lambda(1-u) = \mu v^{\eta}(us)^{1-\eta}$. Due to the positive response of search intensity to an increase in productivity, the TB curve shifts left (see Figure 3). The intersection of the two curves gives the equilibrium level of unemployment and vacancies. The shift in the TB curve, along with the increase in labor market tightness, imply that search effort amplifies the effects of a productivity change on unemployment, and has an ambiguous effect on vacancies. The numerical results below show that search effort amplifies the volatility of vacancies as well. This means that under a reasonable calibration, the effect of the shift in the TB curve on vacancies is dominated by the shift in the job creation curve. In summary, adding worker search effort amplifies the responses of labor market tightness, vacancies and the unemployment rate to a permanent change in productivity.

6. Business cycle properties

This section establishes the business cycle properties of the model.

6.1. Calibration

The length of the time period is a quarter of a month, which will be referred to as a week. The discount factor β is set to $1/1.04^{1/48}$, a value consistent with an annual interest rate of 4%. The separation rate is set to that in Shimer (2005); normalizing it to a weekly frequency, $\lambda = 0.0083$. The productivity process $G(p'|p)$ is approximated by a five-state Markov chain using the method of Rouwenhorst (1995).[7] The following targets for the productivity process are taken from Hagedorn and Manovskii (2008): the quarterly autocorrelation of 0.765, and the standard deviation of 0.013 for the HP-filtered productivity process with a smoothing parameter of 1600. At a weekly frequency, these targets require $\varrho = 0.9903$ and $\sigma = 0.0033$.

Normalization

Following Shimer (2005), the target for the mean vacancy-unemployment ratio is 1. Then, the queue length, q, is 1 in steady state. Recall that productivity, p, has been normalized to 1 at the steady state. Then, (12) and (13) provide the following two parametric restrictions:

$$(1-\eta)k = \eta\chi\gamma \tag{20}$$

and

$$z = 1 - \frac{(1-\beta(1-\lambda))\chi\gamma}{\beta(1-\eta)\mu} - \chi(\gamma-1). \tag{21}$$

[7] Galindev and Lkhagvasuren (2010) show that for highly persistent autoregressive processes, the method of Rouwenhorst (1995) outperforms other commonly-used discretization methods.

Given the rest of the parameters, the parameters k and z are chosen to satisfy (20) and (21). The value of μ, the scaling parameter in the matching function, is chosen by targeting an average unemployment rate of 5.7% (Shimer, 2005).

The elasticity of matches to vacancies

The key parameter of the matching technology is the elasticity of matches with respect to vacancies, $\epsilon_{M,v} = \frac{\partial \ln M}{\partial \ln v}$. When search intensity is fixed, this elasticity is given by η, the exponent on vacancies in the matching function. However, when search intensity is allowed to vary, the measured elasticity of matches to vacancies, $\epsilon_{M,v}$, differs from η. Specifically, combining (12) with (16) and (20) gives $s^{\gamma} = \theta$. Given the uniqueness result in Proposition 1, total search intensity is simply $S = us$ where u denotes unemployment. These results imply that, under variable search intensity, the equilibrium number of matches is given by

$$M = \mu v^{1-(1-\eta)(1-1/\gamma)} u^{(1-\eta)(1-1/\gamma)}. \tag{22}$$

At this point, there are two important conclusions. First, the property that the matching function is constant returns to scale with respect to unemployment and vacancies is preserved under variable search intensity. This result is consistent with the fact that empirical studies do not reject constant returns to scale in the matching functions; see the survey of Petrongolo and Pissarides (2001). Second, under endogenous job search effort, the implied elasticity of matches with respect to vacancies is

$$\epsilon_{M,v} = 1 - (1-\eta)\left(1 - 1/\gamma\right). \tag{23}$$

Given the value of γ, η is chosen such that $\epsilon_{M,v} = 0.544$, an elasticity estimate obtained by Mortensen and Nagypál (2007).

Search cost parameters

The curvature parameter of the search cost, γ is set to 2, a value consistent with the empirical literature; see Yashiv (2000), Christensen et al. (2005), and Lise (2013). This is also roughly the value calibrated by Nakajima (2012). The value of χ, the scale parameter of the search cost, is chosen to satisfy $z - \chi = 0.71$, which gives a flow value of unemployment of 71% of productivity; see Hall and Milgrom (2008) for a justification of this value. The benchmark parameter values are reported in Table 3.

Table 3 here.

6.2. Benchmark model results

As shown in Table 4 the benchmark model accounts for nearly 40% of the observed volatility of the vacancy-unemployment ratio, unemployment, and vacancies. Search intensity is procyclical with a standard deviation of 4.9%.

Table 4 here.

As a further test of the model, we evaluate its prediction for the effect of an increase in UI benefits on the duration of unemployment. There is a large micro-labor literature estimating this effect. The bulk of the evidence says that a 10% increase in benefits increases the average duration of unemployment spells by 0.5 to 1.5 weeks (see, for example, Meyer, 1990). The benchmark model predicts that, in response to a 10% increase in benefits, the average duration of unemployment increases by roughly 1 week – in the middle of the range cited above. As Hagedorn et al. (2013) point out, micro studies on the impact of benefits ignore the equilibrium effect on job creation and thus underestimate the impact. While this effect may affect the numbers above, the model's prediction for the impact of UI benefits on unemployment duration are reasonable, even though this moment was not targeted.

6.3. The net impact of variable search intensity

How much of the success of the model can be attributed to variable search intensity? To answer this question, the model is solved while fixing search intensity. The problems of workers and firms in the model with fixed search intensity are provided in Appendix C.

Two cases are considered. First, the model is solved while fixing search intensity at one and using the same parameter value in the matching function, η, as in the benchmark economy. The parameter μ is recalibrated so that average unemployment remains 5.7% which necessitates recomputing the values of z, χ and k. Table 4 shows that fixing search intensity sharply reduces the volatility of unemployment; its percentage standard deviation falls from 4.8% to 0.3%. The variability of vacancies is less affected by fixed search effort; its standard deviation falls by around 30%. The percentage standard deviation of the vacancy-unemployment ratio falls by over half. Put differently, variable search effort accounts for well over 90% of the model's predicted volatility in unemployment, just under 30% of vacancies variability, and around 55% of that of the vacancy-unemployment ratio.

These results show that approximately 21% ($\simeq \frac{0.098-0.043}{0.264}$) of the observed volatility of the vacancy-unemployment ratio is explained by variable search effort. Search intensity explains roughly 35% ($\simeq \frac{0.048-0.003}{0.129}$) of the volatility of cyclical unemployment, and 11% ($\simeq \frac{0.056-0.040}{0.141}$) of the volatility of vacancies. In other words, search intensity has a much larger impact on the percentage standard deviation of unemployment than vacancies. The implication of these results is that introducing endogenous search effort flattens the Beveridge curve, and as a result unemployment in the model takes on a wider range of values; see Figure 4.

Figure 4 here.

Alternatively, the model is simulated while setting η to 0.544 (its empirical counterpart) and keeping search intensity at one. In this case, fixed search effort leads to a much smaller decline in unemployment volatility and a larger decline in that of vacancies. However, the volatility of the vacancy-unemployment ratio is almost the same as for the first fixed effort experiment. Volatility of labor market variables is roughly half that of the benchmark model.

6.4. Average search intensity

Here, a model-consistent measure of average search intensity is constructed, in much the same way that a measure of aggregate productivity can be obtained by performing a "Solow residual exercise." Recall the matching function $m_t = \mu v_t^{\eta} (s_t u_t)^{1-\eta}$, where m_t is matches (equivalently, new hires) at time t, v_t is vacancies posted by firms, u_t is the level of unemployment, and finally s_t is aggregate search effort. This matching function attributes all changes in matches not due to variation in vacancies or unemployment to changes in average search intensity. Given this observation, two measures of aggregate search effort are constructed. The first, dubbed the Shimer (2005) method, measures changes in search intensity by combining the matching technology with the following well known equation: $u_{t+1} = u_t - m_t + u_t^s$, where u_t^s is short-term unemployment (less than five weeks). The second measure, which will be called the Mortensen and Nagypál (2007) method, employs their proposal to use the empirical Beveridge curve to obtain the job-finding rate, m_t/u_t, via $\hat{f}_t = \frac{\lambda(1-u_t)}{u_t}$ where λ is the separation rate. Then, changes in average search intensity can be captured by $\hat{f}_t^{\frac{1}{1-\eta}} \left(\frac{v_t}{u_t}\right)^{\frac{\eta}{\eta-1}}$.

Set the matching function curvature parameter, η, to 0.080, its value in the benchmark calibration. The separation rate is as reported in Table 3. Figure 5 presents imputed average search intensity for the two methods. While these series are noisy – perhaps owing to the fact that the underlying data are monthly – it is clear that average search effort falls sharply during NBER recessions. In two of the more recent recessions, average search intensity has continued to fall after the "official" end of the recession. Overall, the imputed average search effort series clearly exhibits a countercyclical pattern, falling during recessions and rising gradually during expansions.

Figure 5 here.

Business cycle properties for the Mortensen-Nagypál measure of average search effort are reported in Table 4. The percentage standard deviation of search effort is on par with that of unemployment and vacancies. The benchmark calibration accounts for nearly 40% of the volatility in measured search. This series is also weakly procyclical when the cycle is measured by the correlation with labor productivity. Search effort moves strongly with the conventional measure of labor market conditions, labor-market tightness. The calibrated model also predicts a strong positive correlation between these variables.

7. Implications for the matching technology

Here, further implications of the model for the matching technology are discussed.

7.1. Interdependence of matching and search intensity

When search intensity is fixed, the elasticity of the number of matches with respect to vacancies, $\epsilon_{M,v}$, coincides with the matching technology parameter η: $\epsilon_{M,v} = \eta$. However, under endogenous job search effort, the elasticity is given by $\epsilon_{M,v} = 1-(1-\eta)(1-1/\gamma)$ (see Section 6.1). Consequently, the parameter η can differ substantially from $\epsilon_{M,v}$, the elasticity measured directly from data on cyclical unemployment, vacancies and matches. For example, for the benchmark calibration, $\eta = 0.0880$ and $\epsilon_{M,v} = 0.544$. If one ignores variable search intensity, one would erroneously conclude that a ten percent exogenous increase in vacancies will raise the number of matches by more than 5 percent whereas the actual impact could be less than 1 percent. These results show that the matching technology and the costs of search are intimately related. Estimating the two functions simultaneously requires an equilibrium model with endogenous search effort. This paper offers one such a framework.

7.2. Shifts in the Beveridge curve

Throughout this paper, labor market fluctuations have been modeled as arising due to productivity shocks. However, Mortensen and Nagypál (2007) point out that the correlation between labor productivity and the vacancy-unemployment ratio is less than one-half and emphasize the importance of other omitted driving forces. Consistent with their argument, a sizable fraction of the variation of matches is not explained by shifts in unemployment and vacancies. In this context, variation of matches means overall shifts in the number of matches, which includes both cyclical fluctuations and the trend. The results in this paper suggest that variable search intensity can also account for part of the shifts in matches.

First, as mentioned earlier, endogenous search intensity flattens and stretches the Beveridge curve; see Figure 4. Second, it also makes the Beveridge curve more dispersed or thicker. Notice that these two changes for the Beveridge curve reflect the responses of search intensity to a productivity shock.

There could be other types of shifts as well. For instance, (19) shows that increases in the cost parameters k, χ and γ, reduce equilibrium search intensity. Therefore, in general, the total number of matches is given by

$$M(k, \chi, \gamma, v, u) = A(k, \chi, \gamma)v^{\eta}u^{1-\eta}, \tag{24}$$

where A is a decreasing function of its arguments. So, the number of matches for a given level of unemployment and vacancies can shift with these cost parameters. Therefore, changes in the job search and vacancy costs can also shift the Beveridge curve. These have the following two important implications.

First, Lubik (2011) argues that a negative shock to match efficiency A is consistent with the outward shift of the U.S. Beveridge curve in the aftermath of the Great Recession; also see Elsby et al. (forthcoming). This finding, along with (24), raises the possibility that the above cost parameters may be key to understanding persistently high unemployment despite an increased number of vacancies during the recent recovery.

Second, cross-country data show that there are substantial differences in unemployment across countries. Empirical studies have tended to focus on whether taxes or benefits can explain these cross-country unemployment differences; see, for example, Prescott (2004) and Ljungqvist and Sargent (2006). Time spent on job search also differs substantially across countries. For example, according to Krueger and Mueller (2010), on average unemployed workers spend 41 minutes a day searching for a job in the U.S., compared with just 12 minutes in the average European country. The results in this paper suggest that differences in time spent on job search may account for a substantial part of the cross-country differences in unemployment.

8. Conclusion

The textbook DMP model was modified by adding worker search intensity, allowing workers to directly affect the outcome of their job search over the business cycle. A far more innocuous change, dropping Nash bargaining determination of wages in favor of competitive search, was also introduced. Combining data from the CPS and ATUS, we present new evidence in support of the model's prediction that search effort is procyclical; evidence is also presented showing there is a quantitatively important composition bias (related to recent past wages and hours worked) in average search time over the business cycle.

Greater volatility in unemployment and vacancies can be generated by using a high gross flow income for the unemployed while still maintaining a substantial employment surplus through low utility of the unemployed net of search costs. The benchmark model captures nearly 40% of the volatility in vacancies, unemployment and labor market tightness. In contrast, the standard fixed search effort model captures almost none of the variability in unemployment, around 30% of vacancies variability, and about 15% of that of labor market tightness. These results are summarized, visually, in the Beveridge curve, measured at an annual frequency. Whereas the fixed effort model has a steep Beveridge curve with points tightly clustered along a straight line, the endogenous search effort model exhibits a much flatter, more spread out Beveridge curve. These results collectively suggest that endogenous search effort provides a partial resolution of the Shimer puzzle.

While more elastic search effort can improve the model's performance, the analytical results in this paper show that there are limits to this channel. Specifically, a highly elastic search effort would likely be inconsistent with the data on unemployment and vacancies, and particularly the elasticity of matches with respect to the vacancies-unemployment ratio.

To date, endogenous worker search effort has been largely overlooked when estimating the matching technology, a notable exception being Yashiv (2000). Section 7 showed that this omission can lead to an overestimate, by a factor of 5, of the effects on job matching of an increase in vacancies. This problem is not merely of academic interest since it has implications for public policies aimed at reducing unemployment. The results also suggest that when wages are determined by Nash bargaining, choosing the bargaining power of workers based on an estimate of the matching function alone is premature and cannot always guarantee constrained efficiency.

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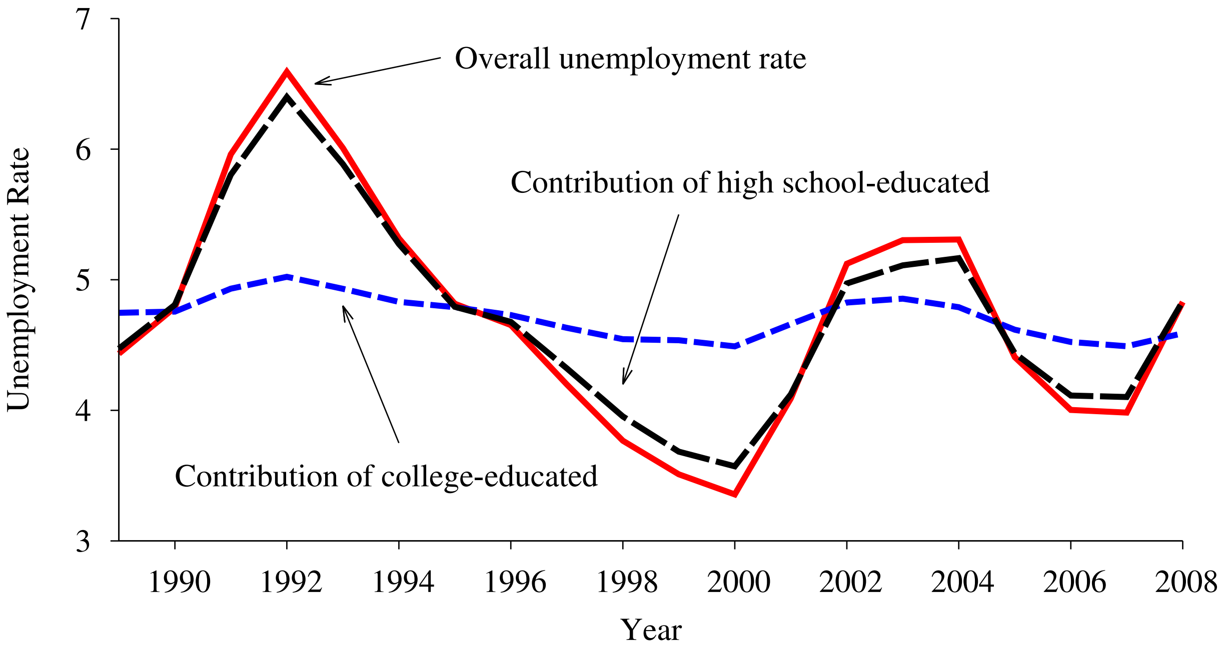
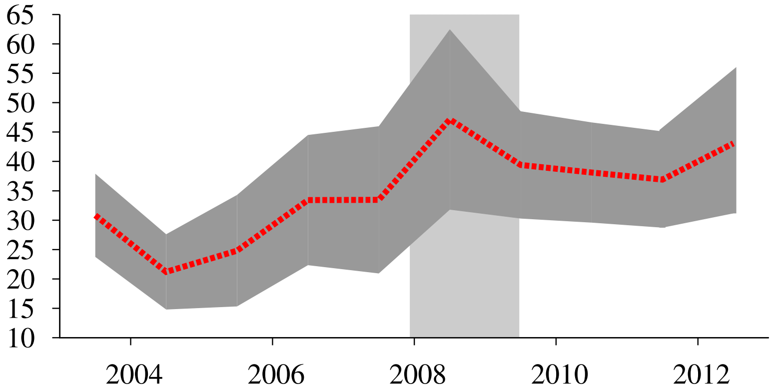
Figure 1: Decomposition of Variation of Aggregate Unemployment



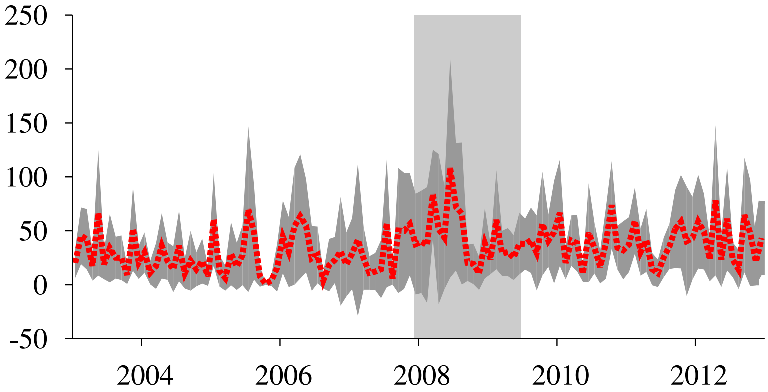
Notes: 'Contribution of college-educated' measures that portion of the cyclical variation in the overall unemployment rate that can be attributed to college educated individuals. Specifically, it computes a hypothetical aggregate unemployment rate that holds the unemployment rate of high school-educated individuals fixed at its sample mean. Similarly, 'Contribution of high school-educated' computes a hypothetical unemployment rate holding the unemployment rate of college-educated at its sample mean. This figure shows that aggregate unemployment fluctuations are mainly driven by unemployment of less educated workers. The coefficients of variation of these two time series over the sample period are 0.035 (contribution of college-educated) and 0.154 (contribution of high school-educated) whereas the coefficient of variation of overall unemployment is 0.182. In other words, unemployment of the less educated group accounts approximately 85% of aggregate unemployment variation over the sample period. The series are constructed from the Current Population Survey of the Bureau of Labor Statistics, which is available from the NBER website. The sample includes adult civilians aged 20-65 years who are in the labor force.

Figure 2: Time Spent on Job Search by the Unemployed



(a) Annual

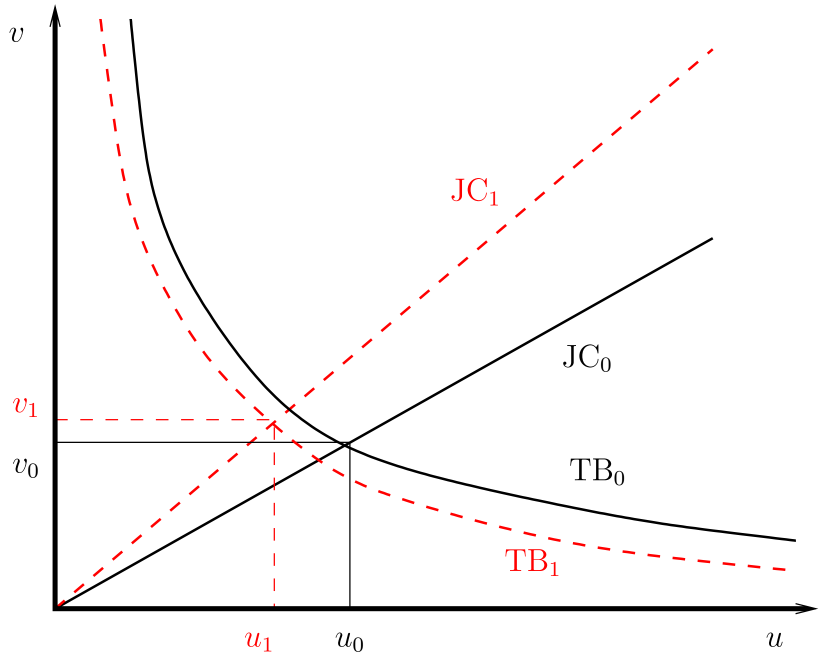
(b) Quarterly



(c) Monthly

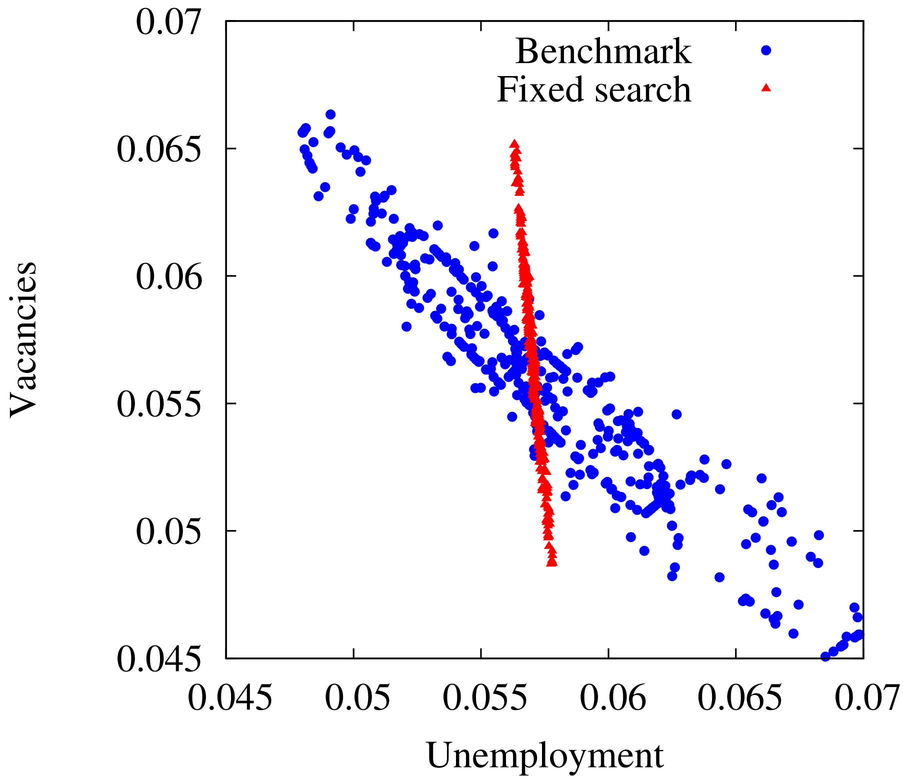
Source: Authors' calculations based on the ATUS. Time on job search is the weighted average of activities corresponding to job search by the unemployed over the relevant time frame. Quarterly and monthly data are constructed using the date of the interview.

Figure 3: The Impact of a Permanent Productivity Change



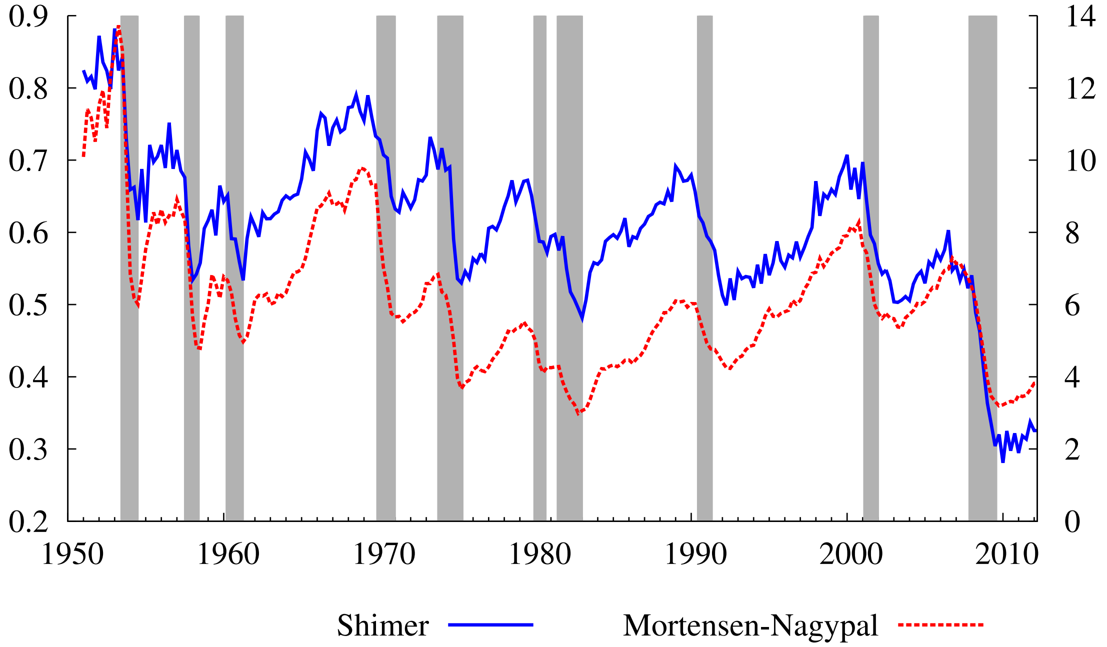
Notes: The figure illustrates how a permanent increase in productivity affects steady state unemployment (u) and vacancies (v). The values denoted by 0 and 1 correspond to values that are before and after the increase.

Figure 4: Beveridge Curves from Model-Generated Data



Notes: This figure shows how variable search effort affects the Beveridge curve in simulated data. It plots the Beveridge curve of the benchmark model and the model with fixed search intensity. A total of 620 annualized observations on unemployment and vacancies have been used.

Figure 5: Average Search Intensity



Notes: "Shimer" corresponds to average search intensity measured using short- and long-term unemployment data (left-hand axis) while "Mortensen-Nagypál" refers to search intensity measured using the empirical Beveridge curve (right-hand axis). See Section 6.4 for the detailed definition of the two measures. Shaded areas are NBER-determined recessions. The two series are unfiltered. Quite similar results are obtained by Hodrick-Prescott filtering the data with a smoothing parameter of 10^5 as in Shimer (2005).

Table 1: Correlation of time spent on job search with unemployment and vacancies

Sample	Unemployment	Vacancies	v-u ratio, θ
All	-0.124	0.297	0.268
	(0.733)	(0.405)	(0.454)
Long-term	0.403	-0.160	-0.226
	(0.248)	(0.656)	(0.531)
Short-term	-0.656^{*}	0.708^{*}	0.716^{*}
	(0.039)	(0.022)	(0.020)
Short-term while controlling for wage and hours	$-0.527^{\dagger}$	0.737^{*}	0.723^{*}
	(0.053)	(0.015)	(0.018)

Notes. This table reports the correlation between average search intensity and labor market variables. Average search intensity is measured by the time dummies in regressions (1) and (2). Significance levels are reported in parenthesis. Correlation coefficients that are significant at the 5% and 10% levels are denoted by an asterisk and a dagger, respectively. To conform with the samples chosen by Shimer (2004) and Mukoyama et al. (2014), data for regression (1) is restricted to adult, civilian, unemployed workers looking for a job, aged 25-70. Data sources for unemployment and vacancies are as in Table 4.

Table 2: Responses of time spent on job search to vacancy-unemployment ratio

sample	The OLS result, a_θ	The implied elasticity, $a_\theta/\overline{s}$
All	3.257	0.142
	(2.912)	
Long-term	-1.605	-0.083
	(2.916)	
Short-term	19.358*	0.516*
	(8.608)	
Short-term while controlling for wage and hours	20.241*	0.540*
	(8.614)	

Notes. This table summarizes the results of the regressions of (3) and (4). The numbers in the left-hand column show the coefficient estimates of a_θ which measures the response of search time to the cyclical deviation of the vacancy-unemployment ratio. The standard errors are reported in parenthesis. The estimates at the significance level of 5% (or less) are denoted by an asterisk. The right-hand column shows the implied elasticity of search time with respect to vacancy-unemployment ratio. Following Krueger and Mueller (2010), the elasticity is calculated as the ratio of the coefficient estimate of a_θ to average search time, $\overline{s}$. The sample restrictions are as in Table 1.

Table 3: Parameters of the Benchmark Model

Parameter	Value	Description
β	0.9992	The time discount factor ($= 1/1.04^{1/48}$)
λ	0.0083	The separation rate ($= 0.1/12$)
ϱ	0.9903	Persistence of the productivity shock
σ	0.0033	The standard deviation of the innovation to productivity
k	0.0261	The vacancy creation cost
z	0.8453	Flow utility of unemployment when search intensity is zero
μ	0.1394	The coefficient of the matching technology
η	0.0880	The parameter of the matching technology
γ	2.0000	The power of the search cost function
χ	0.1353	The average search cost

Notes: Summary of the parameter values used in the benchmark calibration.

Table 4: Select Business Cycle Moments

		u	v	v/u	s	p
US Data:						
Standard deviation		0.129	0.141	0.264	0.128	0.013
Autocorrelation		0.886	0.907	0.905	0.884	0.755
Cross-correlation	u	1	−0.914	−0.976	−0.998	−0.239
	v		1	0.980	0.899	0.381
	v/u			1	0.967	0.320
	s				1	0.173
	p					1
Benchmark Model:						
Standard deviation		0.048	0.056	0.098	0.049	0.013
Autocorrelation		0.828	0.618	0.765	0.765	0.765
Cross-correlation	u	1	−0.788	−0.936	−0.936	−0.934
	v		1	0.955	0.955	0.949
	v/u			1	1	0.996
	s				1	0.996
	p					1
Fixed Effort, benchmark η*:*						
Standard deviation		0.003	0.040	0.043		0.013
Autocorrelation		0.828	0.754	0.765		0.765
Fixed Effort, $\eta = 0.544$*:*						
Standard deviation		0.022	0.026	0.046		0.013
Autocorrelation		0.828	0.619	0.765		0.765

Notes: *US Data:* All moments are based on quarterly data, 1951Q1–2012Q4, logged and HP-filtered with a smoothing parameter of 1600. Unemployment, u, corresponds to the civilian unemployment rate; vacancies are given by a combination of the Conference Board's Help-Wanted Index and work by Barnichon (2010); search effort, s, is computed using the Mortensen-Nagypál method described in Section 6.4; and productivity, p, is measured by output per person for the non-farm business sector (BLS variable PRS85006163). *Models:* Averages over 20,000 replications of the model economy with 248 quarters are reported, after discarding the first 1,000 weeks of data.

Appendix A. Time spent on job search versus the number of search methods

Following Krueger and Mueller (2010) and DeLoach and Kurt (2013), this paper focused on time spent on job search as worker search effort. Others have focused on the number of search methods in the CPS. As discussed in the text, there is considerable debate whether the number of search method is a reasonable measure of search intensity (Shimer, 2004; Tumen, 2014).

This appendix shows that despite the positive link between the two variables at the individual level, their cyclical behavior can be quite different. Before, going to the analysis it should mentioned that in the ATUS time spent on job search and the number of search methods refer to different time periods. Specifically, the former refers to a specific diary day (the day right before the interview date) while the latter refers to the four weeks preceding the diary date. Furthermore, our analysis of the ATUS and CPS data reveals that at the individual level, there is not a great deal of persistence in the number of search methods used. These observations already suggest that the link between the two variables may not be very strong.

First, it is shown that the two variables are positively correlated at the individual level. For this purpose, consider the following regression:

$$s_{i,t} = \tilde{c} + \tilde{\beta}\mathbf{X}_i + \psi n_{i,t} + \epsilon_{i,t} \quad \text{(A.1)}$$

where $s_{i,t}$ is search effort of person i in year t, $\tilde{c}$ is the constant term, $\mathbf{X}_i$ contains the individual characteristics such as age, education, dummies for race and sex, $n_{i,t}$ is the number of search methods and $\epsilon_{i,t}$ is the error term. Using the sample described above, the estimate of ψ is 10.547 with the standard deviation 3.132. Thus, cross-sectionally, a unit increase in the number of job search methods is associated with more than a 10 minute increase in job search time. Despite this highly significant, positive relationship, the R^2 of the regression is approximately 0.084 implying that less than 10 percent of the variation of job search time

Table A.5: Correlation of the number of average search methods with unemployment and vacancies

	Unemployment	Vacancies	The v-u ratio, θ
All	0.448	−0.313	−0.369
	(0.194)	(0.379)	(0.295)
Long-term	0.539	−0.348	−0.426
	(0.108)	(0.324)	(0.220)
Short-term	0.180	−0.212	−0.204
	(0.619)	(0.557)	(0.572)

Notes. This table reports the correlation of unemployment and vacancies with the average number of search methods (after controlling for age, education, race and sex). The significance levels are in parenthesis.

is explained by the regression.

Next it is shown that despite the positive link between the two variables, they behave quite differently over the business cycles. For this purpose, (1) is estimated while considering the number of search methods, $n_{i,t}$, as the left hand side variable. The correlation of the time dummies with labor market variables is reported in Table A.5. Comparing Tables 1 and A.5 reveals that the cyclical pattern of average time spent on job search and the average number of job search methods respond differently to aggregate labor market conditions. Specifically, the sign of the correlation coefficients are vastly different. For example, the number of job search methods responds to labor market tightness negatively, while job search time tends to respond positively, especially among the short-term unemployed. Moreover, the correlation between the number of job search methods with the labor market variables is stronger among the long-term unemployed, whereas the correlation between time spent on job search with the same variables are stronger among the short-term unemployed.

Appendix B. Model with variable search intensity

Appendix B.1. The definition of the labor market equilibrium

Since unemployed workers are intrinsically identical, it follows that $U(p)$ is common to all unemployed workers. Further, $\tilde{U}(\tilde{w}, p)$ must be the same for all jobs for which workers actually search. It then follows that the queue length, $q_{\tilde{w},p}$, must be unique for all jobs with positive worker search: The compensation for searching for a lower wage job is a higher probability of being matched, that is, a lower queue length. Using (5) and (6), it can be seen that search intensity, $s_{\tilde{w},p}$, must also be unique for each job type $(\tilde{w}, p)$. Introducing the following functions, $s(\tilde{w}, p) = s_{\tilde{w},p}$, $q(\tilde{w}, p) = q_{\tilde{w},p}$, $v(\tilde{w}, p) = v_{\tilde{w},p}$, $u(\tilde{w}, p) = u_{\tilde{w},p}$ and $S(\tilde{w}, p) = S_{\tilde{w},p}$ for any $(p, \tilde{w})$ such that $\tilde{w} \in \mathcal{W}(p)$, the labor market equilibrium can now be defined.

Definition 1. *The equilibrium is a set of value functions, $\{U, W, J, V\}$, a decision rule s, a set of the present discounted values of the wages, $\mathcal{W}$, the measures, $\{u, v\}$, the total search intensity, S, and the queue length, q, such that*

1. *unemployed: given q and W, the decision rule $s(\tilde{w}, p)$ and the value functions $U(p)$ and $\tilde{U}(\tilde{w}, p)$ solve (5) and (6) for any $\tilde{w} \in \mathcal{W}(p)$;*
2. *employed: given U, the value function $W(\tilde{w}, p)$ solves (7);*
3. *matched firm: the value function $J(\tilde{w}, p)$ solves (8);*
4. *vacancy: given q and J, the wage $\tilde{w}$ and value function $V(p)$ solve (9) with $\tilde{w} \in \mathcal{W}(p)$;*
5. *free entry: for any real number x,*
$$\begin{cases} v(x,p) > 0 \text{ and } V(p) = 0 & \text{if } x \in \mathcal{W}(p), \\ v(x,p) = 0 \text{ and } V(p) \leq 0 & \text{if } x \notin \mathcal{W}(p) \text{ or } \mathcal{W}(p) = \emptyset; \text{ and} \end{cases} \tag{B.1}$$
6. *consistency: the total search intensity S and the queue length q are consistent with individuals' and firms' behavior: $S(\tilde{w}, p) = u(\tilde{w}, p)s(\tilde{w}, p) = v(\tilde{w}, p)q(\tilde{w}, p)$ for $\tilde{w} \in \mathcal{W}(p)$.*

Appendix B.2. Proof of Proposition 1

Let $Z^e(p) = \int Z(p')dG(p'|p)$ and $R(p) = \displaystyle\int \left(\int Q(p'')dG(p''|p') \right) dG(p'|p)$. Then, (10) can be rewritten as
$$\frac{c'(s_{\tilde{w},p})}{\beta f(q_{\tilde{w},p})} = \tilde{w} + R(p) - U^e(p). \tag{B.2}$$

On the other hand, using the free entry condition,

$$\frac{k}{\beta\alpha(q_{\tilde{w},p})} = -\tilde{w} + Z^e(p). \tag{B.3}$$

Combining (B.2) and (B.3), it can be seen that

$$\frac{c'(s_{\tilde{w},p})}{\beta f(q_{\tilde{w},p})} + \frac{k}{\beta\alpha(q_{\tilde{w},p})} = Z^e(p) + R(p) - U^e(p).$$

Furthermore, using (11),

$$\frac{k}{\beta\eta\alpha(q_{\tilde{w},p})} = Z^e(p) + R(p) - U^e(p).$$

The right hand side of the equation is common across all jobs posted at a given point in time. Since α is a strictly increasing function, $q_{\tilde{w},p}$ is unique across vacancies. Then, the free entry condition in (B.3) implies that $\tilde{w}$ is the same across all vacancies posted at a given point in time.

Appendix B.3. The steady state characterization

When there are no shocks to productivity, i.e. when p is constant over time, a job is fully characterized by its per-period wage $w = (1 - \beta(1 - \lambda))\tilde{w}$. The value of being unemployed is given by

$$U = \max_s \{z - c(s) + \beta f(q)s\big(W - U\big) + \beta U\} \tag{B.4}$$

and the value of being employed is

$$W = \frac{w + \beta\lambda U}{1 - \beta(1 - \lambda)}. \tag{B.5}$$

A worker will take the queue length, q, as given. Differentiating the right hand side of (B.4) with respect to search effort, s, gives

$$c'(s) = \beta f(q)(W - U).$$

Combining this result with (B.4) and (B.5), it can be shown that the optimal search intensity must satisfy the following:

$$w - z = \frac{1 - \beta(1 - \lambda)}{\beta f(q)} c'(s) + c'(s)s - c(s). \tag{B.6}$$

Firms making their vacancy posting decision will take (B.6) as given. The value of a vacancy can be written as

$$V = \max_{w}\{-k + \beta\alpha(q)\frac{p-w}{1-\beta(1-\lambda)}\}. \tag{B.7}$$

Following Rogerson et al. (2005), substitute (B.6) into (B.7) for w and thereby reduce a firm's problem to the following:

$$\max_{q}\left\{\alpha(q)\left(p - z - \frac{1-\beta(1-\lambda)}{\beta f(q)}c'(s) - c'(s)s + c(s)\right)\right\}.$$

Taking the first-order condition with respect to q yields (13).

Appendix B.4. Proof of Proposition 2

Given the inverse relationship between queue length, q, and worker search intensity, s, the right hand side of (13) is strictly increasing in s. Therefore, s increases with productivity, p. A higher s and a lower q means a higher vacancy-unemployment ratio. More vacancies per unemployed worker along with higher search intensity imply a higher job-finding rate.

Appendix B.5. Normalizations

Suppose that search intensity is normalized to $x > 0$. Let the associated search cost function be $\tilde{c}$. Denote the vacancy cost and the coefficient of the matching function by $\tilde{k}$ and $\tilde{\mu}$, respectively. The equilibrium allocations continue to be characterized by (12) and (13). Then, it can be seen that the same allocation is obtained by choosing the cost function to satisfy $\tilde{c}'(x)x - \tilde{c}(x) = c'(1) - c(1) > 0$ while setting $\tilde{k} = \frac{x\tilde{c}'(x)}{c'(1)}k$ and $\tilde{\mu} = \frac{x^{\eta}\tilde{c}'(x)}{c'(1)}\mu$.

As in Shimer (2005), the normalization of θ, the vacancy-unemployment ratio, is inconsequential to the results. Consider another value, say $\overline{\theta}$, for the mean vacancy-unemployment ratio. Then, it can be seen that multiplying k and μ by $\overline{\theta}$ and $\overline{\theta}^{\eta}$, respectively, leaves the equilibrium allocations given by (12) and (13) unaffected.

Appendix B.6. Productivity and the vacancy-unemployment ratio

The implied elasticity of the job-finding rate with respect to the vacancy-unemployment ratio can be written as

$$\tilde{\eta} = \frac{d\ln(f(q)s)}{d\ln\theta} = \frac{d\ln(q\alpha(q)s)}{d\ln\theta} = \frac{d\ln(\theta\alpha(q))}{d\ln\theta} = 1 + \frac{d\ln\alpha(q)}{d\ln\theta}. \tag{B.8}$$

Since $\ln\theta = \ln s - \ln q$, (B.8) can be written as

$$\tilde{\eta} - 1 = \frac{\epsilon_{q,s}}{1-\epsilon_{q,s}}\frac{d\ln\alpha(q)}{d\ln q}, \tag{B.9}$$

where $\epsilon_{q,s} = \frac{d\ln q}{d\ln s}$. Recalling that $\theta = s/q$, differentiation of (11) gives $\epsilon_{q,s} = -\frac{sc''(s)}{c'(s)}$ in equilibrium. Differentiate $\ln\theta = \ln s - \ln q$ with respect to $\ln p$ to obtain the elasticity of the vacancy-unemployment ratio θ with respect to productivity p:

$$\frac{d\ln\theta}{d\ln p} = (1-\epsilon_{q,s})\frac{d\ln s}{d\ln p}. \tag{B.10}$$

As in Section 5.1, let $s = 1$. Then, by taking logs in (13) and differentiating the result with respect to $\ln p$, it can be shown that

$$\frac{d\ln s}{d\ln p} = \frac{p}{p-z} \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})}\frac{c'(1)}{c''(1)+c'(1)} + \frac{c'(1)-c(1)}{c''(1)}}{\frac{1-\beta(1-\lambda)}{\beta f(q)}+1}. \tag{B.11}$$

Now combining (B.10) and (B.11) along with $\epsilon_{q,s} = -\frac{c''(1)}{c'(1)}$, one can arrive at

$$\frac{d\ln\theta}{d\ln p} = \frac{p}{p-z} \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})} + \left(1-\frac{c(1)}{c'(1)}\right)\left(1+\frac{c'(1)}{c''(1)}\right)}{\frac{1-\beta(1-\lambda)}{\beta f(q)}+1}. \tag{B.12}$$

Appendix B.7. Elasticity of the profit with respect to productivity

Combining the free entry condition $k = \beta\alpha(q)\frac{p-w}{1-\beta(1-\lambda)}$ with (B.11) and (B.12), the elasticity of a firm's profit with respect to productivity is given by

$$\frac{d\ln(p-w)}{d\ln p} = \frac{p}{p-z} \times (1-\tilde{\eta}) \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})} + \left(1-\frac{c(1)}{c'(1)}\right)\left(1+\frac{c'(1)}{c''(1)}\right)}{\frac{1-\beta(1-\lambda)}{\beta f(q)}+1}. \tag{B.13}$$

When $c(s) = \chi s^{\gamma}$, this equation is further simplified to

$$\frac{d\ln(p-w)}{d\ln p} = \frac{p}{p-z} \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)}+1-\tilde{\eta}}{\frac{1-\beta(1-\lambda)}{\beta f(q)}+1}. \tag{B.14}$$

Comparing this result with the corresponding expression when search is constant, (C.16), profits are more sensitive to productivity in the model with endogenous search intensity than that in the model with fixed search intensity. Specifically, using our calibrated values, it can be seen that the elasticity is 70% higher in the model with variable search intensity. So, the wage moves less in the model with fixed search intensity due the effects discussed in Section 5.2.

Appendix C. Model with fixed search intensity

Appendix C.1. Workers

When search intensity is fixed at one, the flow utility of unemployment becomes

$$\tilde{z} = z - c(1).$$

Then, the value of being unemployed is given by

$$U(p) = \tilde{z} + \beta f(q) \left[\mathbb{E}_p W(w, p') - \mathbb{E}_p U(p')\right] + \beta \mathbb{E}_p U(p'). \tag{C.1}$$

The value of being employed is as before:

$$W(w, p) = w + \beta(1-\lambda)\mathbb{E}_p W(w, p') + \beta\lambda \mathbb{E}_p U(p'). \tag{C.2}$$

Given U and Q, let

$$H(p) = \mathbb{E}_p[\mathbb{E}_{p'} Q(p'')] - \mathbb{E}_p U(p'). \tag{C.3}$$

Then, (C.1) can be written as

$$U(p) = \tilde{z} + \beta f(q) \left(\frac{w}{1-\beta(1-\lambda)} + H(p) \right) + \beta \mathbb{E}_p U(p'). \tag{C.4}$$

Therefore, for any posted wage $w \in \mathcal{W}(p)$,

$$\frac{w}{1-\beta(1-\lambda)} + H(p) = \frac{U(p) - \tilde{z} - \beta \mathbb{E}_p U(p')}{\beta f(q)}. \tag{C.5}$$

Appendix C.2. Firms

As in Rogerson et al. (2005), substituting (C.5) into (9) for w and taking the first order condition with respect to q yields

$$\frac{y(p)}{1-\beta(1-\lambda)} + H(p) = \frac{U(p) - \tilde{z} - \beta \mathbb{E}_p U(p')}{\beta \alpha'(q)}. \tag{C.6}$$

Combine (C.5) and (C.6) to obtain

$$\frac{y(p) - w}{1-\beta(1-\lambda)} = \frac{\eta}{\mu\beta(1-\eta)} \left[U(p) - \tilde{z} - \beta \mathbb{E}_p U(p')\right] q^{\eta}. \tag{C.7}$$

Combining this result with the free entry condition,

$$\frac{1-\eta}{\eta} k = \left[U(p) - \tilde{z} - \beta \mathbb{E}_p U(p')\right] q. \tag{C.8}$$

Appendix C.3. Elasticity of the vacancy-unemployment ratio with respect to productivity

In the absence of aggregate shocks, the value of Q simplifies to

$$Q = \frac{\beta\lambda}{1-\beta(1-\lambda)} U. \tag{C.9}$$

Therefore, (C.3) becomes

$$H = -\frac{1-\beta}{1-\beta(1-\lambda)} U. \tag{C.10}$$

Then, using these equations, the equilibrium conditions given by (C.6) and (C.8) can be rewritten as

$$\frac{p - (1-\beta)U}{1-\beta(1-\lambda)} = \frac{(1-\beta)U - [z - c(1)]}{\beta\alpha'(q)} \tag{C.11}$$

and

$$\frac{1-\eta}{\eta}\frac{k}{q} = (1-\beta)U - [z - c(1)], \tag{C.12}$$

respectively. Note that (C.11) uses the fact that $y(p) = p$ under a permanent shock. Combining these two equations and using $q = 1/\theta$, one can arrive at

$$p - [z - c(1)] = \frac{1-\eta}{\eta} k \left[\theta + \frac{1-\beta(1-\lambda)}{\beta\mu(1-\eta)} \theta^{1-\eta}\right]. \tag{C.13}$$

As before, by taking logs and differentiating the result with respect to $\ln p$ while taking into account the steady-state normalization $\theta = 1$ and the fact that $\tilde{\eta} = \eta$,

$$\epsilon^F_{\theta,p} = \frac{d\ln\theta}{d\ln p} = \frac{p}{p - [z - c(1)]} \times \frac{\frac{1}{1-\tilde{\eta}}\frac{1-\beta(1-\lambda)}{\beta\mu} + 1}{\frac{1-\beta(1-\lambda)}{\beta\mu} + 1}. \tag{C.14}$$

Given the normalizations $s = 1$ and $q = 1$, $\mu = f(q)$. Thus,

$$\epsilon^F_{\theta,p} = \frac{p}{p - [z - c(1)]} \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})} + 1}{\frac{1-\beta(1-\lambda)}{\beta f(q)} + 1}. \tag{C.15}$$

Appendix C.4. Elasticity of the profit with respect to productivity

Combining the free entry condition $k = \beta\alpha(q)\frac{p-w}{1-\beta(1-\lambda)}$ with (C.15), the elasticity of a firm's profit with respect to productivity is given by

$$\frac{d\ln(p - w^F)}{d\ln p} = \frac{p}{p - [z - c(1)]} \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)} + 1 - \tilde{\eta}}{\frac{1-\beta(1-\lambda)}{\beta f(q)} + 1} \tag{C.16}$$

This elasticity is smaller than the one found in (B.14) (also see the discussions at the end of Appendix B.7). Using (C.16), it can also be seen that a higher elasticity of the number of matches with respect to vacancies, $\tilde{\eta}$, implies a less volatile profit and, thus, a more volatile wage.